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BOOK OF PROBLEMS ON THE STRUCTURAL MECHANICS OF  
SHIPS (SELECTED PARTS)

• Sh. Z. Loktsina

Foreign Technology Division  
Wright-Patterson Air Force Base, Ohio

17 December 1975

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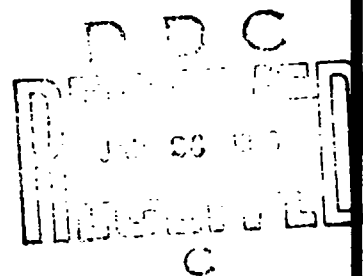
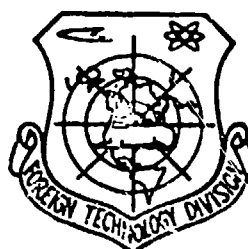
## FOREIGN TECHNOLOGY DIVISION



BOOK OF PROBLEMS ON THE STRUCTURAL MECHANICS  
OF SHIPS (SELECTED PARTS)

by

Sh. Z. Loktsina



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# U. S. BOARD ON GEOGRAPHIC NAMES transliteration SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	<i>А а</i>	A, a	Р р	<i>Р р</i>	R, r
Б б	<i>Б б</i>	B, b	С с	<i>С с</i>	S, s
В в	<i>В в</i>	V, v	Т т	<i>Т т</i>	T, t
Г г	<i>Г г</i>	G, g	У у	<i>У у</i>	U, u
Д д	<i>Д д</i>	D, d	Ф ф	<i>Ф ф</i>	F, f
Е е	<i>Е е</i>	Ye, ye; E, e*	Х х	<i>Х х</i>	Kh, kh
Ж ж	<i>Ж ж</i>	Zh, zh	Ц ц	<i>Ц ц</i>	Ts, ts
З з	<i>З з</i>	Z, z	Ч ч	<i>Ч ч</i>	Ch, ch
И и	<i>И и</i>	I, i	Ш ш	<i>Ш ш</i>	Sh, sh
Й й	<i>Й й</i>	Y, y	Щ щ	<i>Щ щ</i>	Shch, shch
К к	<i>К к</i>	K, k	Ъ ъ	<i>Ъ ъ</i>	"
Л л	<i>Л л</i>	L, l	Ы ы	<i>Ы ы</i>	Y, y
М м	<i>М м</i>	M, m	Ь ь	<i>Ь ь</i>	'
Н н	<i>Н н</i>	N, n	Э э	<i>Э э</i>	E, e
О о	<i>О о</i>	O, o	Ю ю	<i>Ю ю</i>	Yu, yu
П п	<i>П п</i>	P, p	Я я	<i>Я я</i>	Ya, ya

\*ye initially, after vowels, and after ъ, ь; e elsewhere.  
 When written as ё in Russian, transliterate as yë or è.  
 The use of diacritical marks is preferred, but such marks  
 may be omitted when expediency dictates.

## GREEK ALPHABET

Alpha	Α α	•	Nu	Ν ν
Beta	Β β		Xi	Ξ ξ
Gamma	Γ γ		Omicron	Ο ο
Delta	Δ δ		Pi	Π π
Epsilon	Ε ε	•	Rho	Ρ ρ ϱ
Zeta	Ζ ζ		Sigma	Σ σ ς
Eta	Η η		Tau	Τ τ
Theta	Θ θ	•	Upsilon	Υ υ
Iota	Ι ι		Phi	Φ φ ϕ
Kappa	Κ κ	•	Chi	Χ χ
Lambda	Λ λ		Psi	Ψ ψ
Mu	Μ μ		Omega	Ω ω

# RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	$\operatorname{sech}^{-1}$
arc csch	$\operatorname{csch}^{-1}$
<hr/>	
rot	curl
lg	log

## GRAPHICS DISCLAIMER

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## CHAPTER 1.

### BENDING OF STRAIGHT SHAFTS AND THE SIMPLEST SHAFT SYSTEMS

#### Brief Theoretical Information

The differential equation for the bending of beams with variable cross sections which support a distributed load of intensity  $q(x)$  has the form

$$[EI(x)w_1']' = q(x), \quad (1.1)$$

where  $E$  is the modulus of normal elasticity of the beam material;  $I$  is the moment of inertia of the cross-sectional area of the beam. The integral of this equation can be written in the form

$$w_1(x) = \int_0^x \int_0^x \frac{1}{EI(\xi)} \int_0^\xi q(x) dx d\xi d\xi + \\ + N_0 \int_0^x \int_0^x \frac{x}{EI(\xi)} dx d\xi + M_0 \int_0^x \frac{dx d\xi}{EI(\xi)} + \theta_0 x + f_0, \quad (1.2)$$

where  $N_0$ ,  $M_0$ ,  $\theta_0$  and  $f_0$  are the shear force, bending moment, angle of rotation and bending point of the beam on its cross section with coordinate  $x=0$ , respectively, determined from the boundary conditions for fastening the end sections of the beam.

The elastic line for knife-edge beams which support distributed load  $q(x)$  is determined by the equation

$$w_1(x) = \frac{1}{EI} \int_0^x \int_0^x \int_0^x q(x) (dx)^3 + \frac{N_0 x^3}{6EI} + \frac{M_0 x^2}{2EI} + \theta_0 x + l_0. \quad (1.3)$$

If the knife-edge beam is loaded by concentrated force P and moment M, as well as by a distributed load which has a different law of change in different sections along the beam, the elastic line of the beam can be determined by the initial parameters method, using the expression

$$\begin{aligned} w_1(x) = & \frac{N_0 x^3}{6EI} + \frac{M_0 x^2}{2EI} + \theta_0 x + l_0 + \frac{1}{EI} \int_0^x \int_0^x \int_0^x q_1(x) (dx)^3 + \\ & + \int_{x>a_1} \frac{P(x-a_1)^2}{6EI} + \int_{x>a_2} \frac{M(x-a_2)^2}{2EI} + \\ & + \int_{x>a_3} \frac{1}{EI} \int_{a_1}^x \int_{a_2}^x \int_{a_3}^x |q_2(x) - q_1(x)| (dx)^3, \end{aligned} \quad (1.4)$$

where  $a_1$ ,  $a_2$  and  $a_3$  are the cross-sectional coordinates where concentrated force P and moment M are applied;  $q_1(x)$  and  $q_2(x)$  are the load on the section along the beam  $0 \leq x \leq a_3$  and  $a_3 \leq x \leq l$ , respectively ( $l$  is the length of the beam).

If concentrated moments are not applied to the beam, its elastic line is found from shear  $w_2(x)$  by integrating the equation

$$w_2'(x) = -\frac{N(x)}{G\omega} = -\frac{EI w_1''(x)}{G\omega}, \quad (1.5)$$

where G is the shear modulus;  $\omega$  is the area of the cross section of the beam wall, hence

$$w_2(x) = -\frac{M(x)}{G\omega} = -\frac{EI w_1'(x)}{G\omega} + \text{const.} \quad (1.6)$$

When the beam load is made up of concentrated moments, bending moment  $M(x)$  should signify that portion of the total

bending moment which acts in cross section  $x$  of the beam, which is created by the external load, with the exception of the concentrated moments applied, when determining the elastic line from shear according to formula (1.6). The concentrated moments do not cause displacements from shear and impart only an additional rotation which is determined by the expression  $\varphi = -\frac{1}{G_{\text{cut}}} \sum_{i=1}^n M_i$ , where  $M_i$  is the  $i$ -th concentrated moment acting on the beam, as a result of the shearing strain of the entire cross section of the beam.

The integration constants in expressions (1.2)-(1.4) should be determined from the condition for fastening the support sections of the beam. So, if the support sections of the beam ( $x=0$ ) and ( $x=l$ ) are fixed elastically on elastic supports, the boundary conditions are written in the form

$$\left. \begin{aligned} x=0 \quad w_1 + w_2 &= -A_1 [EI(x) w_1]'; \quad w_1' = \alpha_1 E/w_1; \\ x=l \quad w_1 + w_2 &= A_2 [EI(x) w_1]'; \quad w_1' = -\alpha_2 E/w_1, \end{aligned} \right\} \quad (1.7)$$

where  $A_1$  and  $A_2$  are the pliability coefficients of the elastic supports;  $\alpha_1, \alpha_2$  are the pliability coefficients of the elastic fixings, which can be expressed by the coefficients of the support pair which are the ratio of the support moment for the elastic fixing of the beam ends  $M_{\text{sup}}$  to the moment in the support section of the same beam for rigid fixing of its ends  $M_{\text{rig}}$ :  $\alpha = \frac{M_{\text{sup}}}{M_{\text{rig}}}$ .

For a knife-edge beam, when  $\alpha_1 = \alpha_2 = \alpha$  and the load acting on the beam is symmetrical relative to the middle of its length, the coefficient of the support pair does not depend on the value or the nature of the change in load and it is determined from the formula

$$\alpha = \frac{1}{1 + \frac{2\alpha_1 EI}{l}}. \quad (1.8)$$

The bending elements of the beams are the linear functions of the support pair coefficient.

Various methods are used to disclose the static uncertainty of continuous beams. If a continuous beam rests freely on intermediate supports, it is advisable to take the moments on the supports as the fundamental unknown forces (forces method). The angles of rotation of the support sections are used as the fundamental unknowns for continuous beams which are elastically fastened onto intermediate supports (strain method).

If the support moments are used as the basic unknowns in order to determine the static uncertainty of a continuous beam, the condition of the equality of the rotation angles of sections on a common support for two adjacent beam spans stressed by the given external load and unknown support moments should be used to compose the system of equations which determine these moments. When elastic supports are present, sagging of these supports are also taken as the unknowns along with the support moments. In this case, it is necessary to compose additional equations to solve the problem, the equations obtained from examining the equilibrium state of elastic supports being advisable for this purpose.

In this case the system of equations will be the following: conditions for the equality of the rotation angles at the  $j$ -th support

$$\frac{M_{j-1}l_j}{6EI_j} + \frac{M_j l_j}{3EI_j} + \alpha_j(Q_j) + \frac{l_j - l_{j-1}}{l_j} = -\frac{M_j l_{j+1}}{3EI_{j+1}} - \frac{M_{j+1} l_{j+1}}{6EI_{j+1}} + \alpha_j(Q_{j+1}) + \frac{l_{j+1} - l_j}{l_{j+1}}. \quad (1.9)$$

The condition of the equilibrium of the  $j$ -th elastic support

$$l_j = A_j \left[ \frac{M_j - M_{j-1}}{l_j} + \frac{M_j - M_{j+1}}{l_{j+1}} + \frac{Q_j c_j}{l_j} + \frac{Q_{j+1} (l_{j+1} - c_{j+1})}{l_{j+1}} \right], \quad (1.10)$$

where  $l_j$ ,  $l_{j+1}$  are the length of the span between  $(j - 1)$  and the  $j$ -th support and between the  $j$ -th and the  $(j + 1)$  support,

respectively;  $Q_j, Q_{j+1}$  is the external load acting on spans  $(j-1), j$  and  $j, (j+1)$ , respectively;  $\alpha_j(Q_j), \alpha_j(Q_{j+1})$  are the angles of rotation of the section on the  $j$ -th support from the load in span  $(j-1), j$  and  $j, (j+1)$ , respectively;  $M_{j-1}, M_j, M_{j+1}$  are the support moments on supports  $(j-1), j, (j+1)$ , respectively;  $f_{j-1}, f_j, f_{j+1}$  are the sagging of supports  $(j-1), j, (j+1)$ , respectively;  $A_j$  is the pliability coefficient of the  $j$ -th elastic support;  $I_j$  and  $I_{j+1}$  are the moments of inertia of the cross-sectional area of the beam in spans  $(j-1), j$  and  $j, (j+1)$ , respectively;  $c_j$  and  $c_{j+1}$  are the distances of equivalent loads  $Q_j$  and  $Q_{j+1}$ , respectively, from a support placed to the left of them.

The condition of the equilibrium of separate joints of the beam should be used compose the system of equations determining the angles of rotation of the support sections if these angles are used as the unknowns. The dependence between the angles of rotation of the support sections and the support moments when elastic supports are present can be written in the form

$$M_{ij} = \boxed{M_{ij}} - \frac{2EI_{ij}}{l_{ij}} \left[ 2\alpha_i + \alpha_j - 3\left(\frac{l_j - l_i}{l_{ij}}\right) \right], \quad (1.11)$$

where  $M_{ij}$  is the support moment at the  $i$ -th support of beam span  $i-j$ ;  $\boxed{M_{ij}}$  is the moment on the  $i$ -th support from the load acting on span  $i-j$  with the assumption of the total fixing of the end sections of this span;  $\alpha_i$  and  $\alpha_j$  are the rotation angles of the section on the  $i$ -th and  $j$ -th supports, respectively;  $f_i$  and  $f_j$  are the sagging of the  $i$ -th and  $j$ -th supports. Then the system of equations obtained from the condition of the equilibrium of the moments applied to the  $i$ -th joint (support) will be

$$2\alpha_i \sum_j \frac{2EI_{ij}}{l_{ij}} + \sum_j \frac{2EI_{ij}}{l_{ij}} \left( \alpha_j - 3\frac{l_j - l_i}{l_{ij}} \right) = \sum_j \boxed{M_{ij}} + \mathfrak{M}_i, \quad (1.12)$$

where  $\mathfrak{M}_i$  is the external moment which acts on the  $i$ -th joint.

Additional equations should be written in form (1.10).

The moments in the joints and the displacement of mobile joints (if there are any) are taken as the unknowns when designing simple plane assemblies composed of straight shafts. The equations which determine the static uncertainty of these frames are composed by equating the angles of rotation of the shaft sections at their common joints according to system (1.9). The principle of potential displacements can be used to compose additional equations. In this case, the sum of the work of all the external forces and the moments of the joints for potential displacements are equated to zero (assuming that hinges are mounted on the joints).

When designing complex assemblies, i. e., frames in which more than two shafts can converge at the joints, it is advisable to use the angles of the joints' rotation from the load and the angles of obliquity (if mobile joints are present) as the unknowns. The equations which determine the static uncertainty of complex assemblies are obtained from the equilibrium equations of the frame joints. Additional equations are composed for assemblies with mobile joints and rectangular floors on the basis of the principle of potential displacements.

The kinematic relationships which relate the angles of obliquity of the shafts to each other should be used to reduce the number of unknown angles of obliquity of complex frames. Complex assemblies can also be designed by using the method of the successive balancing of joints.

When designing simple and complex assemblies with symmetrical construction, it is advisable to make use of the advantages of symmetrical construction no matter which method is used. For this purpose, each unsymmetrical load is separated into symmetrical and antisymmetrical and the frame is calculated for each load separately.

## Problems

### Bending of Single-Span Beams<sup>1</sup>

1. Find the elastic line of a knife-edge beam (cantilever), the left end of which ( $x=0$ ) is elastically fastened (pliability coefficient  $\alpha$ ) to a rigid support and the right end of which ( $x=l$ ) is completely free. The intensity of the load on the beam  $q(x) = q_0 \frac{x}{l}$ .

2. Find the elastic line of a knife-edge beam, the left end of which ( $x=0$ ) is elastically fastened (pliability coefficient  $\alpha$ ) and the right ( $x=l$ ) - resting freely on a rigid support. The load intensity  $q = \text{const}$ .

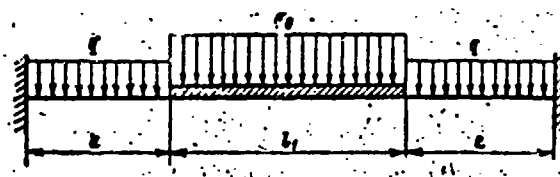


Fig. 1.

3. Determine the elastic line of a knife-edge beam in the section  $x=0$  which is supported by a hinge on an elastic support with a pliability coefficient  $A$ , and in section  $x=l$  - rigidly fastened. The load intensity  $q = \text{const}$ .

4. Determine the support moments as well as the elastic line of a knife-edge beam which is fastened elastically at the ends (pliability coefficient  $\alpha$ ) and stressed in the middle of the span by force  $P$ .

<sup>1</sup>The moment of inertia of the cross-sectional area of knife-edge single-span beams is taken as equal to  $I$  in all the problems. The origin of the coordinates is taken at the left end of the beam for single-span beams; the span length  $l$  is taken as equal to  $l$ .

5. Determine the values of the pliability coefficient of elastic fixing  $\alpha$  at which the greatest bending moment in the span of a knife-edge beam which is freely supported at one end and elastically fastened at the other and which is uniformly loaded with a distributed load of intensity  $q$  will be equal in absolute value to the bending moment in the fastening. Calculate the greatest bending moment in the span.

6. Determine the support bending moments as well as the greatest sagging of a beam which is fixed elastically at the ends, the middle portion of which, with length  $2l_1$ , is absolutely rigid (Fig. 1).  $a$  is the length of the end sections of the beam. The pliability coefficient of the elastic fastening of the beam's ends is  $\alpha$ . The intensity of the load acting on the middle part of the beam is  $q_0$  and that on the end portions -  $q$ .

7. Given the equation for the bending moment of a freely supported knife-edge beam which is loaded with concentrated force  $P$  in section  $x=c$ :

$$M = -Pl \left[ \frac{(l-c)x}{l^3} - \frac{(x-c)^2}{2l} \right].$$

Determine the equation of the bending moment for the same beam, but stressed in section  $x=c$  with concentrated moment  $M_0$ , using the superposition method.

8. Knowing the expression for the angles of rotation of a freely supported knife-edge beam loaded with concentrated force  $P$  in section  $x=c$

$$\theta'(x) = \frac{Pl^3}{6EI} \left\{ \frac{(l-c)}{l} \left[ 1 - \frac{(l-c)^2}{l^2} - \frac{3x^2}{l^3} \right] + 3 \left( \frac{x-c}{l} \right)^2 \right\},$$

determine the expression for the angles of rotation of the same beam, but uniformly loaded with distributed load of intensity  $q$ , by the superposition method.

9. Using the superposition method, determine the elastic line of a freely supported knife-edge beam which is under the action of a load which varies according to the law  $q(x) = q_0 \frac{x}{l}$ . If the elastic line of the same beam loaded with concentrated force  $P$  applied to section  $x=c$  is known:

$$w(x) = \frac{Pl^3}{6EI} \left\{ \frac{(l-c)}{l} \cdot \frac{x}{l} \left[ 1 - \frac{(l-c)^3}{l^3} - \frac{x^3}{l^3} \right] + \left( \frac{x-c}{l} \right)^3 \right\}.$$

10. Determine the elastic line of a cantilever with a staggered cross section which is loaded on the free end with concentrated force  $P$  and the moments of inertia on the cross-sectional area of the separate sections of the beam are  $I_1$  and  $I_2$  (Fig. 2).

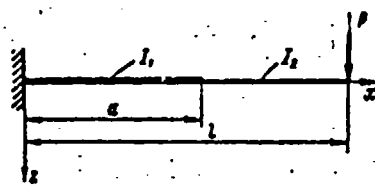


Fig. 2.

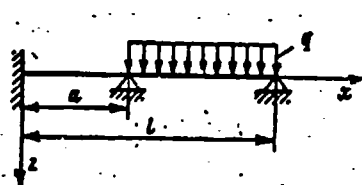


Fig. 3.

11\*. Using the initial parameters method, determine the elastic line from bending of the knife-edge beam shown in Fig. 3. The intensity of the evenly distributed load is  $q$ .

12. Construct the bending moment and shear diagrams of a beam with a staggered cross section which is evenly loaded by a distributed load of intensity  $q$  (Fig. 4).

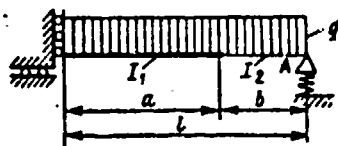


Fig. 4.

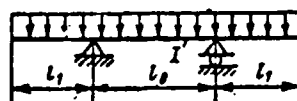


Fig. 5.

13. Determine the ratio of lengths of sections of a knife-edge beam ( $\gamma = I_1/I_2$ ) at which: a) the bending moment in the cross sections above the supports is equal to the moment in the middle section of the beam; b) the bending indicators of the ends of the cantilever are equal to zero (Fig. 5).

14. Establish the values of the support pair coefficients  $\alpha_1$  and  $\alpha_2$  for the support sections of a beam which is fastened elastically at the ends and which is stressed with a load which varies according to the law  $q(x) = q_0 \frac{x}{l}$  (Fig. 6).

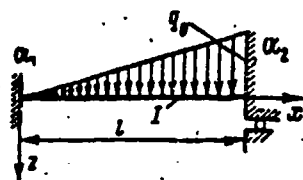


Fig. 6.



Fig. 7.

15. Determine the bending moments in the support sections of an elastically fixed beam which is loaded with a uniformly distributed load of intensity  $q$  (Fig. 7).

16. Using the solution to problem 15, determine the bending moments in the fastenings and in the middle support for the beams shown in Fig. 8.

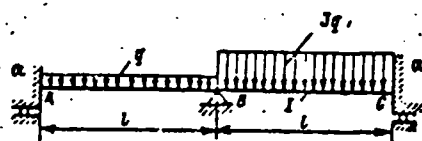


Fig. 8.

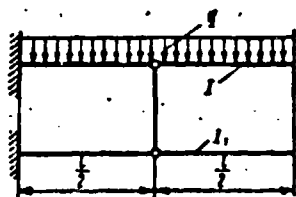


Fig. 9.

17. For a knife-edge beam, one end of which ( $x=0$ ) is rigidly fastened and the other ( $x=l$ ), freely supported on a rigid support, determine the cross section in which concentrated moment  $M_0$  should be applied so that the sagging of this section is equal to zero.

18. Two beams which are rigidly fixed at the ends are connected to each other by an incompressible spacer (Fig. 9) which is fastened to them by a hinge. At what ratio of the moments of inertia of these beams ( $I_1/I$ ) does the stressed beams' support moment decreases two times relative to the moment without the spacer?

19. Solve problem 18 with the assumption that the beams are joined by two incompressible spacers which are fastened by hinges and placed at distance  $l/3$  from the support and from each other.

20\*. Compose the differential equation of bending for a system of two identical beams (Fig. 10) which are joined by incompressible spacers which work during bending. There are a rather large number of spacers. Also find the equation for the elastic line of the beams when they are resting freely on rigid supports.

21. Disregarding the effect of sagging on the change in support forces, determine the greatest sagging for two cases of stress for a floating knife-edge beam: 1) the beam is stressed in the middle of its length by concentrated force  $P$ ; 2) concentrated forces  $P/2$  are applied at the end sections of the beam.

22. Find the expression for the elastic line of a freely resting knife-edge beam loaded in cross section  $c$  by concentrated force  $P$  with consideration of the effect of shear strain. The area of the wall cross section is  $\omega$ .

23. A freely resting knife-edge beam with wall cross-sectional area  $\omega$  is stressed as shown in Fig. 11. Determine the elastic line of this beam with consideration of shear.

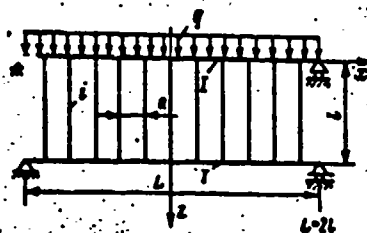


Fig. 10.

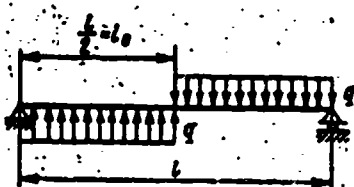


Fig. 11.

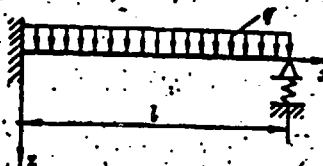


Fig. 12.

24. Find the elastic line and bending moment in any cross section of the knife-edge beam shown in Fig. 12, with consideration of the effect of tangential stress on its bending. The cross-sectional area of the beam's wall is equal to  $\omega$ . The pliability coefficient of the elastic support is  $A$ .

25. Find the equation of the elastic line of a rigidly fastened knife-edge beam which is loaded with a uniformly distributed load of intensity  $q$  with consideration of shear. Take  $\frac{E\theta}{G\omega} = 0.04$ . Find the ratio of the beam's sagging in the middle of its length with consideration of shear and without consideration of shear.

26. Determine the sagging in the middle of the length of a knife-edge beam which is elastically fixed on rigid supports due to shear and bending for two load situations: 1) a uniformly distributed load of intensity  $q$ ; 2) concentrated force  $P$ , applied in the middle of the beam's length. The coefficient of the support pair of the fastening of the beam's end sections is  $\kappa$ .

27. Determine the reaction  $R$  of the interaction of two intersecting beams (Fig. 13) with consideration of sagging due to shear.

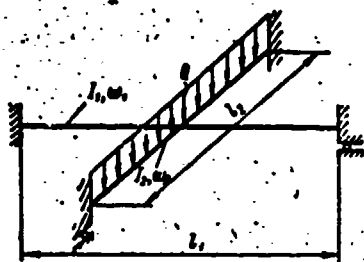


Fig. 13.



Fig. 14.

28. Determine the support bending moments with consideration of shear strain for three cases of bending of knife-edge beams (cross-sectional area of walls is  $\omega$ ): 1) one end of the beam is resting freely and the other is rigidly fixed; bending moment  $M_0$  is applied to the freely supported end; 2) the beam is rigidly fixed at the ends and is stressed with a load which changes according to the law  $q(x) = q \frac{x}{l}$ ; the origin of the coordinates is taken in the support cross section of the beam; 3) the beam is rigidly fastened at the ends; one of the support sections received displacement  $f$ .

29. Determine the maximum sagging of a rigidly fixed knife-edge beam (Fig. 14) which is stressed with a distributed load which varies according to the law  $q = \frac{q_1}{2} \left(1 - \cos \frac{2\pi x}{l}\right)$ , with consideration of shear strain.

#### Bending of Continuous Beams

30. Determine the pliability coefficients of the elastic supports of the beam at points C and D (Fig. 15).

31\*. Determine the pliability coefficients of elastically fixed beam AB, shown in Fig. 16.

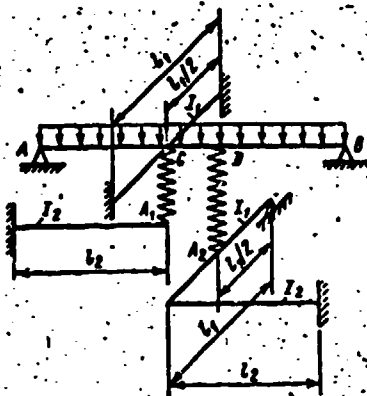


Fig. 15.

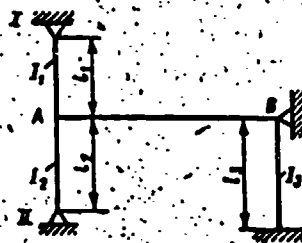


Fig. 16.

32. Determine the bending moment in the support cross sections and in the middle of a two-span beam which is fastened symmetrically at the ends, is loaded by a uniformly distributed load and rests on an elastic support placed in the middle of the beam's length (Fig. 17).

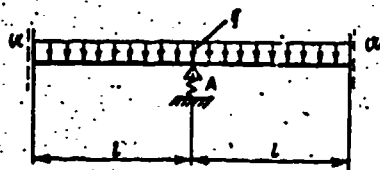


Fig. 17.

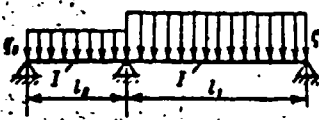


Fig. 18.

33. Determine the ratio between the intensity of loads  $q_1$  and  $q_2$  at which the angle of rotation at the middle support of a knife-edge beam (Fig. 18) will be equal to zero;  $l_1/l_2 = 1.25$ .

34. Determine the vertical displacement of hinge G as well as the settling of elastic support F of a multi-span knife-edge beam (Fig. 19). The pliability coefficient of the elastic support  $A = l^3/48EI$ . Construct the bending moment and shear diagrams.

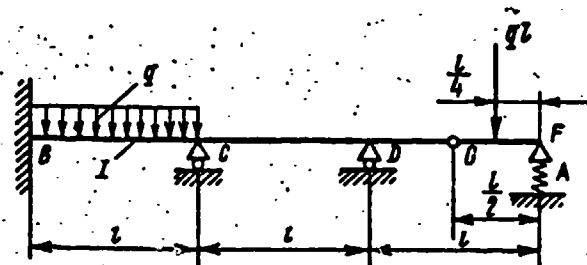


Fig. 19.

35. A bridge is resting on three pontoons. Find the dependence between the moment of inertia of the bridge  $I = \text{const}$  and the water line areas  $F_1$  and  $F_2$  of the pontoon if the settling of the pontoons  $f_1, f_2$  are related by dependence  $\frac{f_1}{f_2} = \frac{F_1}{F_2}$  under the effect of concentrated force  $P$  (Fig. 20). The specific gravity of water is  $\gamma$ .

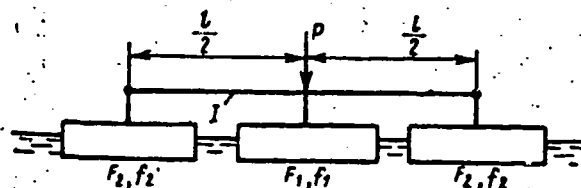


Fig. 20.

36. A continuous knife-edge beam which is resting on a rigid shore support and on the edge of an absolutely rigid weightless straight-walled pontoon is loaded according to Fig. 21. Find the bending moment in the beam at  $x=2l$ . The pontoon is joined to the beam by hinges. Let the length of the pontoon  $L = \frac{EI}{\gamma b^3}$ , where  $\gamma$  is the specific gravity of water,  $b = ?$  is the pontoon's width.

37. Determine the value of the pliability coefficient  $A$  of the elastic supports of a knife-edge beam (Fig. 22) at which the calculated value of the bending moment will be equal to  $M = \frac{q l^2}{8}$ .

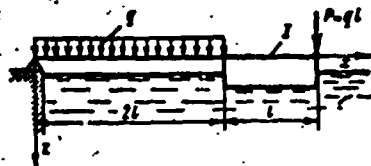


Fig. 21.

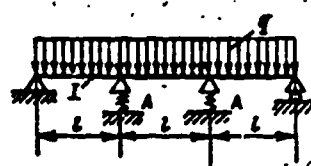


Fig. 22.

38\*. Construct the bending moment and shear diagrams for a continuous beam with a constant cross section (Fig. 23).

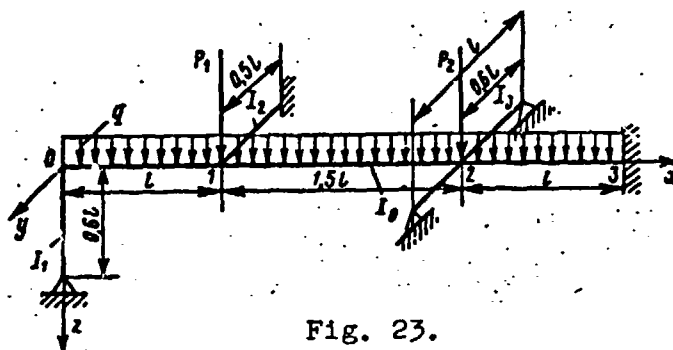


Fig. 23.

Given:  $I_1 = 6I_0$ ;  $I_2 = 2.08I_0$ ;  $I_3 = 1.92I_0$ ;  $P_1 = 0.5ql$ ;  $P_2 = ql$ , where  $I_0$  is a constant value with the dimensionality of the moment of inertia.

39. Determine the static uncertainty of the continuous beams shown in Figures 24 and 25. Construct the bending moment and shear diagrams. In Fig. 24 the concentrated force  $P = 0.187 ql$  and the load intensity to the right is  $0.6 q$ .

40. Discover the static uncertainty of the continuous beam shown in Fig. 26. Given:  $P = 2ql$ ;  $Q_1 = 3ql$ ;  $Q_2 = ql$ ;  $M_0 = 0.1ql^2$ ;  $I_1 = I_2 = I_3 = 2I$ ;  $I_4 = 4I$ ;  $I_5 = I$ . Find the values of the support bending moments,

the moments in the middle of the span and the shear forces at the ends of the separate spans of the beam.

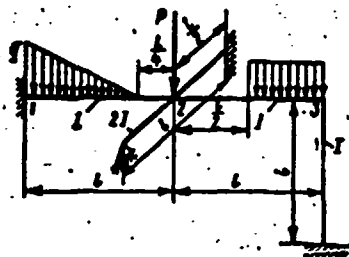


Fig. 24.

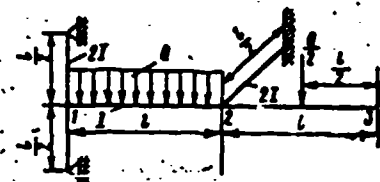


Fig. 25.

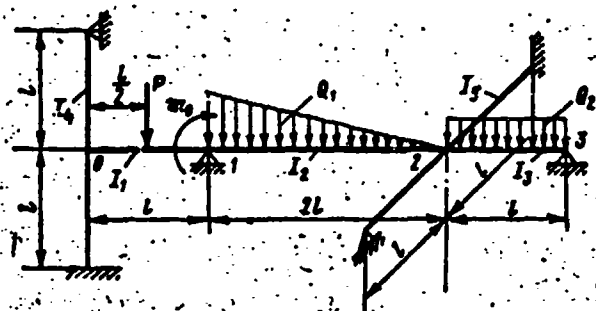


Fig. 26.

Construct the bending moment and shear diagrams.

41. Construct the bending moment and shear diagrams for the continuous beam shown in Fig. 27.

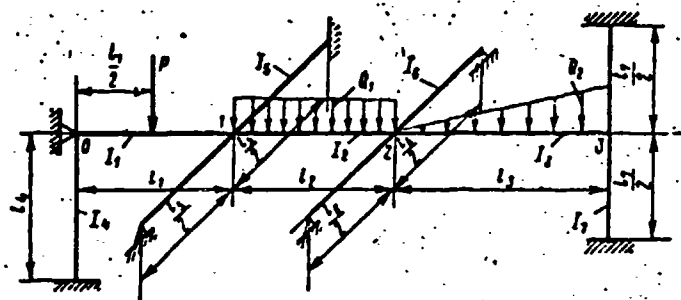


Fig. 27.

Given:  $P = 2ql$ ;  $Q_1 = 1.5ql$ ;  $Q_2 = 2ql$ ;  $I_1 = I_2 = I_3 = I$ ;  $I_4 = I_5 = I_7 = 1.5I$ ;  
 $I_6 = 0.75I$ ;  $I_1 = I_2 = I_3 = I_4 = 2I_5$ ;  $I_6 = 3I_5$ ;  $I_8 = I_7 = I_9$ .

42. Determine the bending moments in the support cross sections and the settling of the elastic support of the continuous beam shown in Fig. 28  $\Delta_1 = l/3EI_0$ ;  $A_1 = l^2/48EI_0$ .

43. Determine the angle of rotation of the cross section of the continuous beam shown in Fig. 29, stressed by moment  $M_0$ .

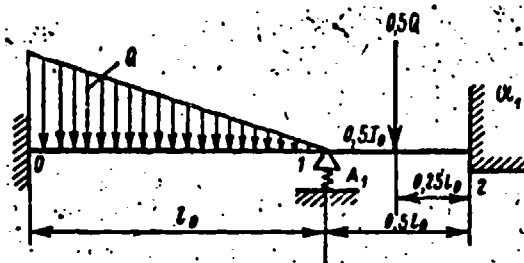


Fig. 28.



Fig. 29.

44. Determine the bending moments  $M_0$  and  $M_1$  at  $x = l/2$  in the knife-edge beams shown in Fig. 30. The spacer is considered to be incompressible and is joined to the beams by hinges. The moments of inertia of the cross-sectional area of the beams are  $I_0$  and  $I_1$ .

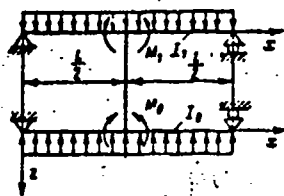


Fig. 30.

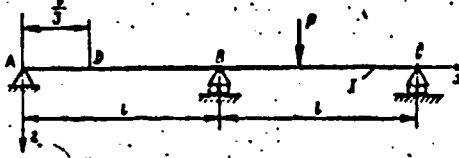


Fig. 31.

45. For a two-span knife-edge beam (Fig. 31) it is necessary to determine the position of force  $P$  which acts in span  $BC$  at which the bending moment in section  $x = l/3$  has the greatest value.

46. Determine the ratio between lines  $l_1$  and  $l_2$  of a knife-edge beam (Fig. 32) at which the sagging at point  $A$  returns to zero.

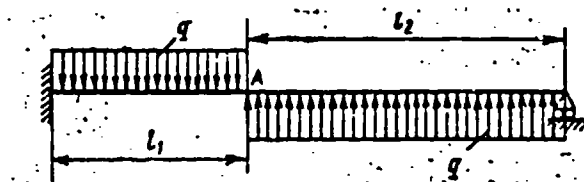


Fig. 32.

47. By how much must the middle support of a knife-edge beam (Fig. 33) be lowered so that the bending moment in the section above this support vanishes?

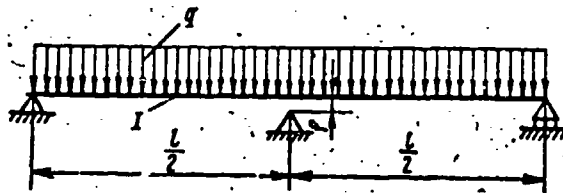


Fig. 33.

48. Find the support moments in a rigidly fixed beam with a staggered cross section (Fig. 34).

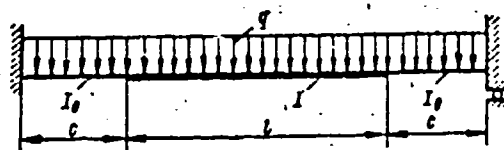


Fig. 34.

49. Determine the value of a span of a continuous knife-edge beam (Fig. 35) at which the angle of rotation of the right support section under the action of concentrated moment  $M$  will comprise 0.9 the angle of rotation of this section when the left section of the beam is absent<sup>+</sup>.

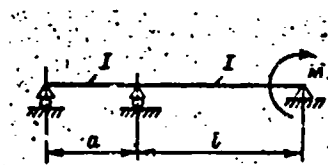


Fig. 35.

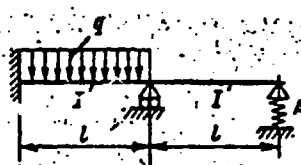


Fig. 36.

50. Find the dependence between the pliability coefficient  $t$  of an elastic support  $A$  and the bending moment in the section in the middle of the stressed span of the knife-edge beam shown in Fig. 36.

51. There is a freely resting knife-edge beam which is fixed in the middle of the span by an elastic support. Determine the value of the pliability coefficient of the elastic support  $A$  at which the greatest bending moment in the span of the beam becomes equal in absolute value to the value of the bending moment in the section which coincides with the middle support if: a) the beam is stressed as shown in Fig. 37; b) the beam is stressed according to Fig. 38.

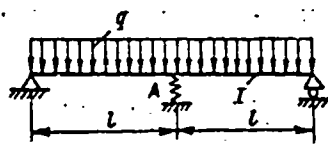


Fig. 37.

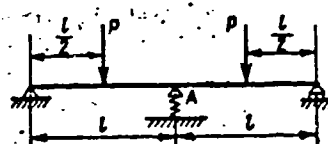


Fig. 38.

52. Construct the bending moment diagram for the knife-edge beam in Fig. 39. Consider the support structure BSTC to be made of absolutely rigid shafts joined by hinges.

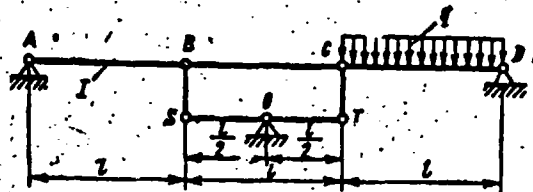


Fig. 39.

53. A bridge is built across a channel in the form of a continuous knife-edge beam which rests on rigid shore supports and the edge of a straight-walled absolutely rigid weightless pontoon. The pontoon is joined to the beam by hinges (Fig. 40). Construct the bending moment diagram for the beam. The pontoon length is  $L = \frac{EI}{\gamma h^3}$ , where  $\gamma$  is the specific gravity of water.

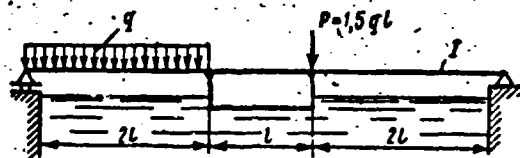


Fig. 40.

54. Determine the mean value of the support pair coefficient<sup>1</sup> for the middle span of a continuous beam (Fig. 41) with the condition that the moments of inertia of the cross-sectional areas of the beams in the adjoining spans are related like the lengths of the same spans, i. e.,  $\frac{I_1}{l_1} = \frac{I_2}{l_2}$ ;  $\frac{I_3}{l_3} = \frac{I_2}{l_2}$ .

55. Determine the support bending moments of a continuous beam (Fig. 42), assuming that the moments of inertia of the cross-sectional areas of the beams in the adjoining spans are related like

<sup>1</sup>The mean value of the support pair coefficient is equal to the ratio of the half-sum of the support moments to the moment of the rigid fastening.

the lengths of these spans, i. e.,  $\frac{l_1}{l_1} = \frac{l_2}{l_2} = \frac{l_3}{l_3}$ . Also compute the mean value of the support pair coefficient for the middle span of the beam.

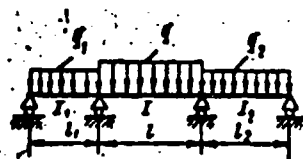


Fig. 41.

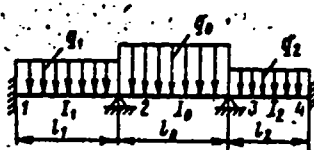


Fig. 42.

56. Discover the static uncertainty and construct the bending moment diagram for a continuous beam with a hinge in the middle span. The beam is stressed with concentrated force  $P$  which is applied to the hinge (Fig. 43). The ends of the beam are fixed elastically (the pliability coefficient of the fastening  $\alpha = \frac{l}{3ET}$ ).



Fig. 43.



Fig. 44.

57. Determine the pliability coefficient of an elastic support arranged in the middle of a freely resting uniformly loaded knife-edge beam of length  $2l$  with the condition that the greatest bending moment in the sections will be four times smaller than in the absence of an intermediate support, i. e., equal to  $ql^2/8$ .

58. When installing a shaft line, the axes of two of its sections were separated by the value  $f$  (Fig. 44). Determine the reactive forces which will be transmitted to supports 2 and 3 when the sections are drawn together at flange A.

59. Determine the separation  $f$  of the axes of two sections of shaft line (Fig. 44) from the condition that when these sections are drawn together at flange A, the reaction on support 1 will be equal to zero. Take the weight of each section of the shaft equal to  $P$ .

#### Designing Assemblies Made of Straight Shafts

60. Find the relative value of counterpressure  $q^*/q$  on the bottom branch of a frame ring (floor) at which the bending moment in the lower end of the ring will become equal to the moment for the rigid fastening of this end (Fig. 45). Given:  $\frac{B}{H} = 1.5$ ,  $\frac{I}{I'} = 15$ .

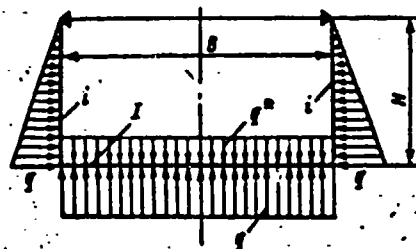


Fig. 45.

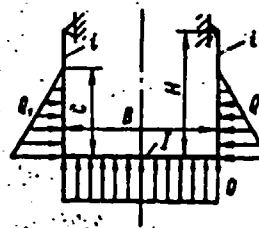


Fig. 46.

61. Determine the support pair coefficient for the floor of a frame assembly (Fig. 46). The ring frame on the upper deck is considered to be resting freely. The moments of inertia of the cross-sectional area of the ring and the floor are equal to  $i$  and  $I$ , respectively.

62. Determine the horizontal displacement of the end of shaft AB, which is rigidly joined at a right angle to shaft BC, if horizontal force  $T$  is applied at point A (Fig. 47). Consider only the bending strain of the shafts.

63. Calculate a simple frame consisting of straight knife-edge shafts (Fig. 48). The shafts of the assembly are joined by

hinger at joints 2 and 2', while joints 3 and 4 are immobile. Construct the bending moment diagrams.

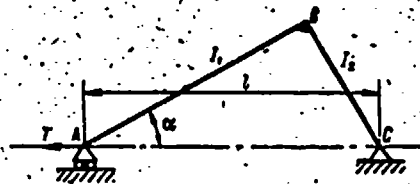


Fig. 47.

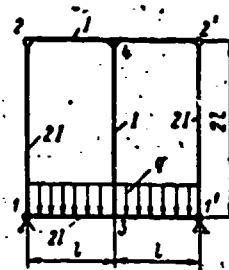


Fig. 48.

64. Considering that the shafts of the frame at joints 2 and 2' are rigidly connected and joints 3 and 4 are immobile, calculate the assembly shown in Fig. 48.

65\*. Determine the bending moments and longitudinal forces in the shafts of the cantilever assembly (Fig. 49) stressed at the end of the cantilever by vertical force P. Consider the dilatational-compressive strain and bending strain of the shafts. The cross-sectional area of the shafts is equal to F, whereupon  $I = 0.5F^2$ ,  $\alpha = 30^\circ$  and  $F = 2 \cdot 10^{-3} l_1^2$ .

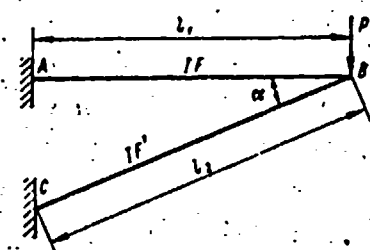


Fig. 49.

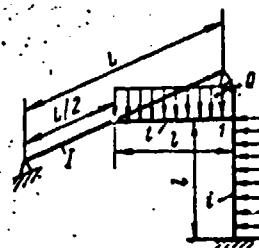


Fig. 50.

66. Find the dependence between the moment of inertia, the length of the beam span supporting the upper shaft of the assembly (Fig. 50) and the bending moment originating at joint 1 of this frame.

67. Determine the bending moments in the joints and the fixing of simple frames with mobile joints 2 and 3 (Figures 51 and 52) with the following data: for the assemblies in Fig. 51,  $l = 8h$ ,  $I_1 = 2I_2 = I_0$ ,  $P = \frac{1}{2} qh$ ; for the assemblies in Fig. 52,  $I_1 = 2I_2$ ,  $I_3 = I_4 = I_0$ .

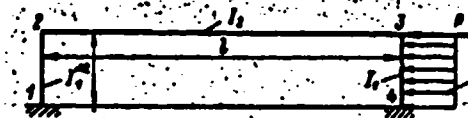


Fig. 51.

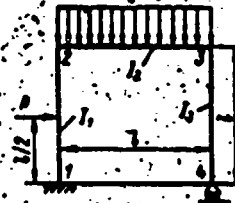


Fig. 52.

68. Determine the bending moments in the joints and construct the bending moment and shear diagrams in the shafts of simple frames with immobile joints (Figures 53 and 54) with the following data: for the frames in Fig. 53,  $h = 5l$ ,  $I_1 = 8I_0$ ,  $I_2 = I_3 = I_0$ ; for the frames in Fig. 54,  $I_1 = 4I_0$ ,  $I_2 = I_3 = I_0$ ,  $A = \frac{l^3}{100EJ_0}$ .

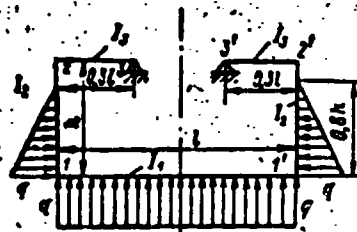


Fig. 53.

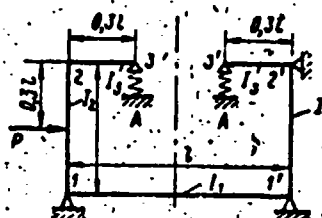


Fig. 54.

69. Determine the moments in the joints of a simple frame with immobile joints and a pillar in the centerplane (Fig. 55) and construct the bending moment and shear diagrams in the shafts of the frame with the following data:  $h = \frac{1}{3} l$ ,  $I_1 = 5I_0$ ,  $I_2 = I_3 = I_0$ .

70. Determine the bending moments in the joints of complex assemblies with immobile joints (Fig. 56, 57, 58 and 59) and construct the bending moment diagrams in the shafts with the following

data: a) for the assemblies in Fig. 56,  $I_1 = 2I_0$ ,  $I_2 = 4I_0$ ,  $I_3 = I_0$ ;  
 b) for the assemblies in Fig. 57,  $I_1 = 8I_0$ ,  $I_2 = 2I_0$ ,  $I_3 = I_0 = 3I_0$ ,  $I_4 = I_0$ ,  $I_5 = 0.5I_0$ ;  
 c) for the assemblies in Fig. 58,  $I_1 = 8I_0$ ,  $I_2 = I_0 = I_0 = 2I_0$ ,  $I_3 = 2I_0$ ,  $I_4 = I_0$ ;  
 for the assemblies in Fig. 59,  $I_1 = 8I_0$ ,  $I_2 = I_0 = 3I_0$ ,  $I_3 = I_0$ ,  $I_4 = 0.5I_0$ ,  $I_5 = 6I_0$ ,  
 $Q_1 = q$ ,  $Q_2 = 0.5q$ .

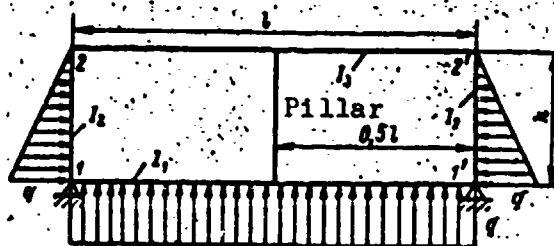


Fig. 55.

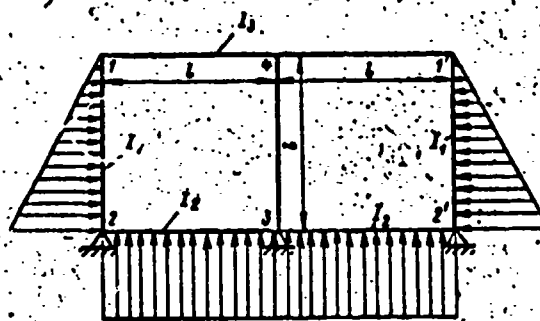


Fig. 56.

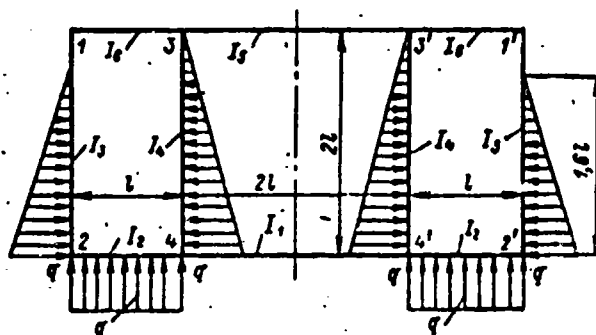


Fig. 57.

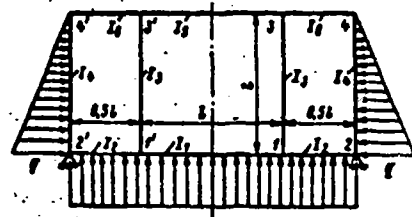


Fig. 58.

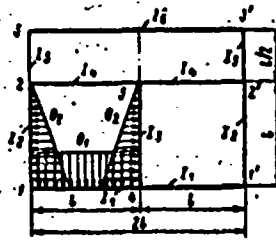


Fig. 59.

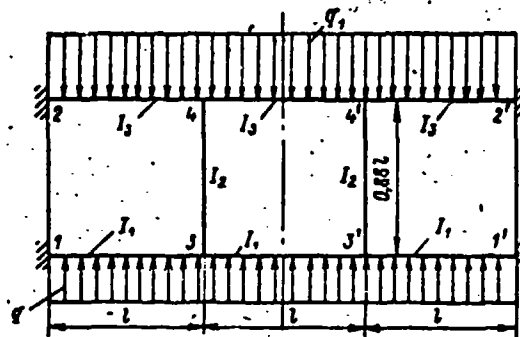


Fig. 60.

71. Determine the bending moments in the joints and fixings of a frame with mobile joints (Fig. 60) with the following data:  $I_1 = I_0$ ;  $I_2 = 0.55I_0$ ;  $I_3 = 3I_0$ . The shafts at joints 1, 1', 2 and 2' are rigidly fixed; joints 3, 3', 4 and 4' are mobile.

72. Calculate the girderless frame with rigid floors (3-4-6-5 and 5'-6'-4'-3') shown in Fig. 61 at  $I_1 = 2I_0$ ;  $I_2 = I_0$ . The shafts at joints 1, 1', 2 and 2' are rigidly fixed.

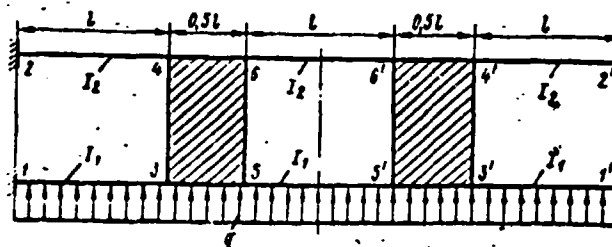


Fig. 61.

73. Determine the moments in the joints of a complex assembly (Fig. 62) with the following data:  $Q_1 = 0.5ql$ ;  $Q_2 = Q_3 = ql$ ;  $I_1 = 4I_0$ ;  $I_5 = I_6 = 2I_0$ ;  $I_4 = I_7 = I_8 = I_9 = I_0$ ;  $I_3 = I_{10} = 0.5I_0$ .

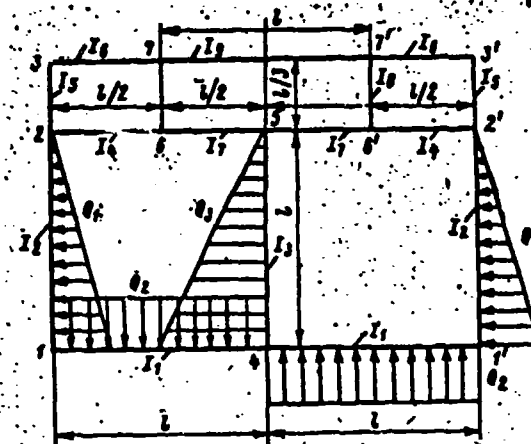


Fig. 62.

## CHAPTER II.

### CALCULATING BEAMS ON AN ELASTIC BASE

#### Brief Theoretical Information

The differential equation for the bending of a beam with a constant cross section which lies on a solid elastic base with constant rigidity has the form

$$EIw^{IV}(x) + km(x) = q(x). \quad (2.1)$$

The common integral of this equation is made up of the common solution to the homogeneous equation corresponding to it [at  $q(x)=0$ ] and the partial solution, depending on the form of the right side. The common integral of the homogeneous equation can be written in one of the following forms:

$$w_{oh} = e^{-\alpha x} (B_1 \cos \alpha x + B_2 \sin \alpha x) + e^{\alpha x} (B_3 \cos \alpha x + B_4 \sin \alpha x); \quad (2.2)$$

$$w_{oh} = C_1 \operatorname{sh} \alpha x \cos \alpha x + C_2 \operatorname{sh} \alpha x \sin \alpha x + C_3 \operatorname{ch} \alpha x \sin \alpha x + C_4 \operatorname{ch} \alpha x \cos \alpha x; \quad (2.3)$$

$$w_{oh} = D_0 V_0(\alpha x) + D_1 V_1(\alpha x) + D_2 V_2(\alpha x) + D_3 V_3(\alpha x); \quad (2.4)$$

$$w_{oh} = E_0 W_0(\alpha x) + E_1 W_1(\alpha x) + E_2 W_2[\alpha(l-x)] + E_3 W_3[\alpha(l-x)]; \quad (2.5)$$

$n \neq j; g \neq h; n = 0, 1, 2, 3; j = 0, 1, 2, 3;$   
 $g = 0, 1, 2, 3; h = 0, 1, 2, 3.$

where  $\alpha = \sqrt{\frac{k}{4EI}}$ ,  $B_i, C_i, D_i, E_i$  are arbitrary integration constants determined from the boundary conditions;  $V_i(\alpha x)$  are the Puzyrevskiy functions;  $W_i(\alpha x)$  are the Klishevich functions.

The values of functions  $V_i(\alpha x)$  and  $W_i(\alpha x)$  are given in [3, Table I] and their properties are given in detail in §15 [1].

If the load intensity  $q(x)$  is a polynomial of no higher than the third power, the partial solution  $w_{\alpha, p}$  is written in the form

$$w_{\alpha, p} = \frac{q(x)}{k} \quad (2.6)$$

When a beam is stressed by concentrated forces, moments or a distributed load which has a different law of change in different sections along the beam, the integral of differential equation (2.1) can be obtained by means of the initial parameters method. In particular, the elastic line for the beams shown in Fig. 63 is written in the form

$$\begin{aligned} w(x) = & D_0 V_0(\alpha x) + D_1 V_1(\alpha x) + D_2 V_2(\alpha x) + D_3 V_3(\alpha x) + \\ & + \int_0^x \frac{P}{2\alpha^3 EI} V_3[\alpha(x-c)] + \int_{c_1}^x \frac{P}{2\sqrt{2}\alpha^3 EI} V_3[\alpha(x-c_1)] + \\ & + \int_{c_2}^x \frac{q}{4\alpha^4 EI} [1 - V_0[\alpha(x-c_2)]] + \int_{c_3}^x \frac{m}{4\alpha^4 EI} [(x-c_3) - \\ & - \frac{1}{\sqrt{2}\alpha} V_1[\alpha(x-c_3)]] \end{aligned} \quad (2.7)$$

where  $m$  is the angular coefficient for a load which varies according to the triangle law.

The Puzyrevskiy functions rapidly increase with the increase in the argument  $\alpha x$ ; therefore, when they are used for numerical computations it is necessary to subtract the small difference of the close values. The Klishevich functions do not have this defect.

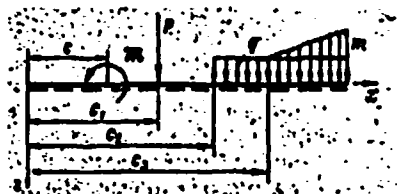


Fig. 63.

The integral of the homogeneous equation for infinitely long beams should be taken in form (2.2), since the condition of the boundedness of sagging at  $x \rightarrow \infty$  is satisfied when constants  $B_3$  and  $B_4$  are equated to zero. The two remaining constants are determined from the

conditions at  $x=0$ .

In beams which lie on a solid elastic base with constant rigidity whose support sections are fastened identically and in which the load acting on the beam is symmetrical relative to the middle of its length, the relationship between the support pair coefficient  $\chi$  and the pliability coefficient of elastic fixing  $\mathcal{K}$  does not depend on the nature of the change in the load and is determined by the formula

$$\chi = \frac{1}{1 + \frac{29EL}{l(\psi_1(u) + 2\psi_0(u))}} \quad (2.8)$$

where  $\psi_1(u)$ ;  $\psi_0(u)$  - are the Bubnov functions for a freely resting beam which lies on an elastic base and which is stressed by a support moment (see appendix VI). The bending elements of the beams are linear functions of the support pair coefficient.

In particular, the solution of the problem of bending of a round cylindrical shell which is stressed by uniformly distributed pressure  $q$  is reduced to the integration of differential equation (2.1). The differential equation which determines the radial displacement of points of the middle surface of this shell has the form

$$Dw^{IV}(x) + \frac{Eh}{R^3}w(x) = q, \quad (2.9)$$

where  $D = \frac{Eh^3}{12(1-\nu^2)}$  is the cylindrical rigidity;  $w(x)$  is the

displacement of the points of the shell in the direction of its radius  $R$  (positive toward the center of curvature);  $h$  is the thickness of the shell.

The integral of the homogeneous equation which corresponds to equation (2.9) is written in one of the forms previously indicated [(2.2), (2.3), (2.4)], whereupon parameter  $\alpha$  is determined from the formula

$$\alpha = \sqrt[4]{\frac{3(1-\nu^2)}{R^3 h^3}}. \quad (2.10)$$

A ship which is floating on calm water is a supportless beam which is acted upon by the force of gravity and lifting forces. The sagging of this ship as an elastic beam is determined from the differential equation in the form

$$[EI(x) w''(x)]' + k(x) w(x) = q(x), \quad (2.11)$$

where  $k(x) = \gamma b(x)$ ;  $q(x) = q_0(x) - \gamma F(x)$ ;  $q_0(x)$  is the intensity of the weight load in cross section  $x$  along the ship;  $F(x)$  is the loaded area of the ring in section  $x$  of an absolutely rigid ship differentiated on calm water;  $\gamma$  is the specific gravity of water;  $b(x)$  is the width of the water line;  $w(x)$  is the sagging of the ship as an elastic beam. Equation (2.11) describes the bending of a nonprismatic beam which lies on a solid elastic base of variable rigidity under the effect of longitudinal distributed force of intensity  $q(x)$ . [Certain of the methods of solving differential equation (2.11) will be given in Chapter IV.]

The differential equation for the bending of a knife-edge beam which lies on a solid base can be written in the form

$$EI w_1^{IV}(x) - k \frac{EI}{G\omega} \dot{w}_1(x) + k w_1(x) = q(x), \quad (2.12)$$

with consideration of shear strain, where  $w_1(x)$  is the sagging of the beam from bending and  $G\omega$  is the beam's rigidity to shear.

Equation (2.12) is analogous to the differential equation for the complex bending of beams which lie on a solid elastic base under the action of tensile force  $T = k \frac{EI}{G_0}$ . If the beam is stressed by a symmetrical load and its support sections are fastened elastically onto rigid supports (pliability coefficient  $\alpha$ ), the boundary conditions have the form at  $x = \pm \frac{l}{2}$ :

$$\begin{aligned} w_1' &= \mp \alpha EI / w_1' \\ w_1 - \frac{EI}{G_0} w_1'' &= w_1 - \frac{M_{en}}{G_0} = 0, \end{aligned} \quad (2.13)$$

where  $M_{en}$  is the value of the bending moment in the support section.

In order to use the available solutions on the complex bending of beams on an elastic base to determine the bending elements of these beams, differential equation (2.12) and boundary conditions (2.13) must be transformed. We will introduce a new function  $v(x)$  which is related to the elastic line from bending  $w_1(x)$  by dependence

$$v(x) = w_1(x) - \frac{M_{en}}{G_0}. \quad (2.14)$$

Then differential equation (2.12) and boundary conditions (2.13) will assume the form, respectively,

$$EI v^{(4)}(x) - k \frac{EI}{G_0} v''(x) + kv(x) = q(x) - \frac{q_0}{G_0} \approx q^*(x); \quad (2.15)$$

at  $x = \pm \frac{l}{2}$

$$\left. \begin{aligned} v' &= \mp \alpha EI / v' \\ v &= 0 \end{aligned} \right\} \quad (2.16)$$

With boundary conditions (2.16), equation (2.15) describes the complex bending of an elastically fastened beam which lies on an elastic base. Professor N. V. Mattes obtained simple formulae for certain of these beams to determine the characteristic bending elements. The numerical values of the functions in these formulae are given in the handbook (Sivertsev, I. N., Davydov V. V., Mattes

N. V. Students' Manual on the Strength of Vessels with Internal Floating. M., 1950) depending on the arguments  $u$  and  $v$ , which are found from the formulae:

$$u = \frac{l}{2} \sqrt{\frac{k}{4EI}}, \quad v = \frac{l}{2} \sqrt{\frac{T}{EI}} = 4u^3 \sqrt{\frac{2(1+\nu)l}{\omega^2}}. \quad (2.17)$$

The necessary functions for beams which are stressed by a uniformly distributed load are given in the tables in appendix VII.

#### Problems

Using the Differential Equation for Bending of Beams on an Elastic Base

74. Find the sagging and angle of rotation at the origin of the coordinates of a semi-infinite knife-edge beam which lies on a solid elastic base with rigidity  $k$  and which is stressed by concentrated force  $P$ . At the origin of the coordinates, the beam is resting on an elastic support with a pliability coefficient

$$A = \frac{1}{4\alpha^3 EI} \quad (\text{Fig. 64}).$$

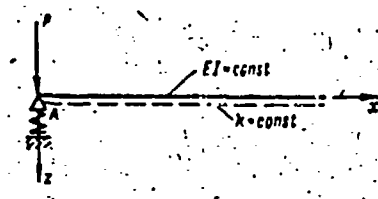


Fig. 64.

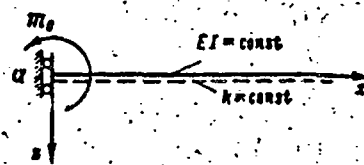


Fig. 65.

75. Find the sagging and angle of rotation at the origin of the coordinates of a semi-infinite knife-edge beam which lies on a solid elastic base with rigidity  $k$  and which is loaded by a concentrated moment. The beam is fixed elastically at the origin

of the coordinates (the pliability coefficient of the elastic fixing  $\alpha = \frac{1}{4aEI}$ ) (Fig. 65).

76. Find the elastic line and the maximum sagging index of an infinite beam which lies on a solid elastic base with rigidity  $k$  and which is loaded by force  $P$ . At the origin of the coordinates, the beam is resting on an elastic support (the pliability coefficient  $\lambda = \frac{1}{16a^3EI}$ ) (Fig. 66).

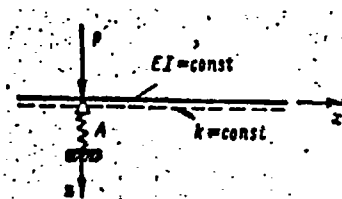


Fig. 66.

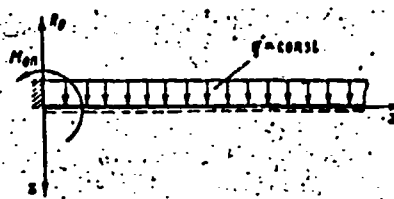


Fig. 67.

77. Find the elastic line, reaction  $R_0$  and support moment  $M_{0n}$  of a semi-infinite beam which is rigidly fixed at the origin of the coordinates, lying on an elastic base and stressed by a uniformly distributed load with intensity  $q$  (Fig. 67).

78. Determine the values of the characteristic bending elements for the knife-edge beams shown in Fig. 68: sagging, angle of rotation, moment and shear in the support sections and in the middle. The rigidity of the elastic base  $k = \frac{64EI}{\pi}$ .

79. Find the greatest sagging of an infinitely long knife-edge beam which lies on a solid elastic base with rigidity  $k$  and which is loaded in a section with a length  $2$  by a uniformly distributed load of intensity  $q$ .

80. Determine the gap  $f$  between the end sections of a semi-infinite knife-edge beam and the elastic support, the support reaction being equal to  $R_0$  (Fig. 69).

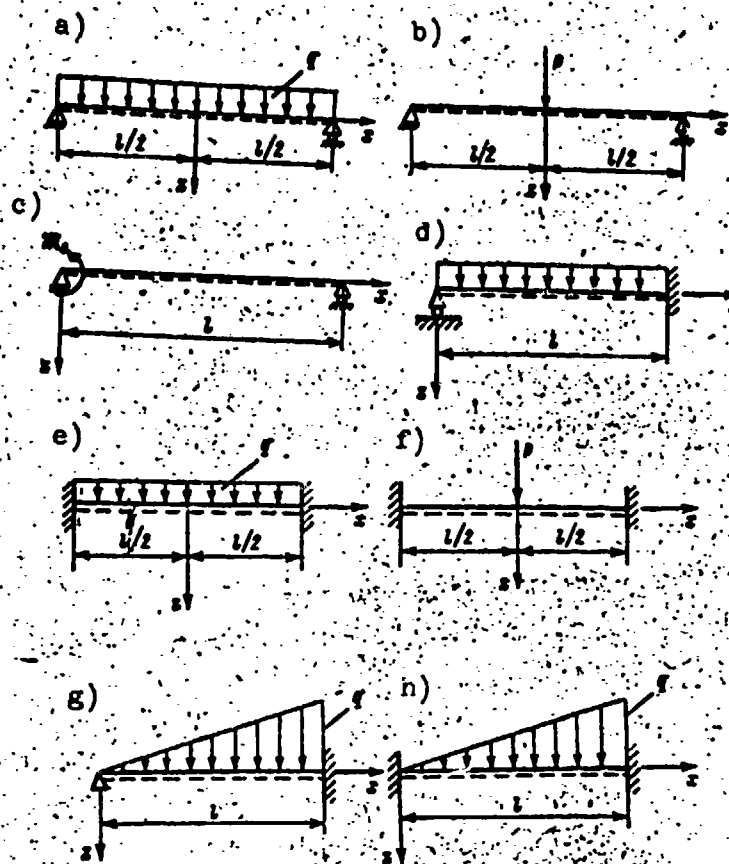


Fig. 68.

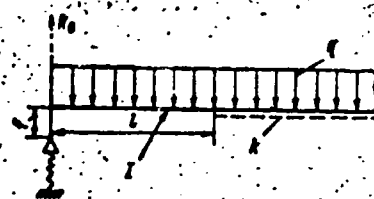


Fig. 69.

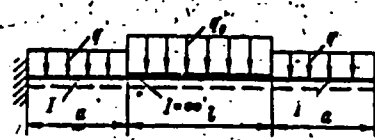


Fig. 70.

81. Determine the support bending moments and sagging in the middle of the length of a fixed beam which lies on a solid elastic base with rigidity  $k$ . The middle portion of the beam is absolutely rigid (Fig. 70).

82. Find the equation for the elastic line of a flexible straight-walled knife-edge pontoon floating on water which is loaded in the middle section with concentrated force  $P$  (Fig. 71). Compute the bending moment acting in the section under the force also. The pontoon's weight is evenly distributed along its length  $l$ , the moment of inertia of the cross-sectional area of the pontoon is  $I$  and the pontoon's width is  $B$ .

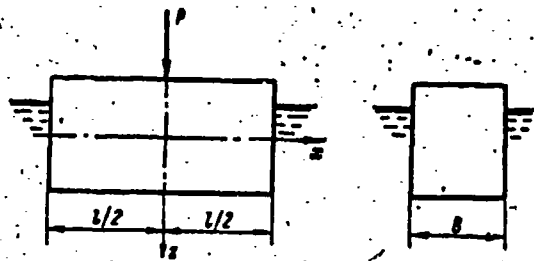


Fig. 71.

83. Find the fibrous bending stresses in an infinitely long round cylindrical shell with radius  $R$  and thickness  $h$  which is stressed by a uniformly distributed lateral load of intensity  $q$  and which is supported in the middle on an undeformable diaphragm. Determine the distance  $a$  to the section of the shell which is closest to the diaphragm in which the bending stress vanishes. The origin of the coordinates is considered to be in the cross section which coincides with the diaphragm.

84. A round infinitely long cylindrical shell with thickness  $h$  and radius  $R$  is stressed on the perimeter of the middle section by a load of intensity  $q$ . Determine the equivalent area  $F$  of a circular ring loaded with a load of intensity  $q$  which receives the same pressure as the shell in the section under the load.

85. Determine the support pair coefficient for the middle span of the continuous beam shown in Fig. 72.

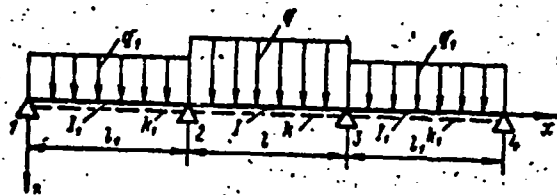


Fig. 72.

86. Solve problem 85 with the assumption that support sections 1 and 4 are rigidly fixed.

87. An infinitely long beam on a solid elastic base is stressed by a uniformly distributed load with intensity  $q$ . A rigid support is installed in a certain section. Determine the reaction of this support and the bending moment in the cross section above the support.

88\*. A knife-edge beam which lies on a solid elastic base is loaded on the ends by two concentrated forces  $Q/2$  (Fig. 73). Considering that the angles of rotation of the end sections are equal to zero, find the bending moments and the reaction of the elastic base in the middle and in the end sections. (This solution can be used to check the strength of the keel when docking ships.)

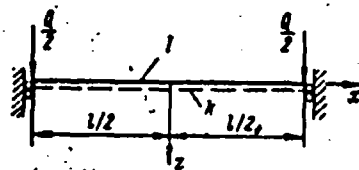


Fig. 73.

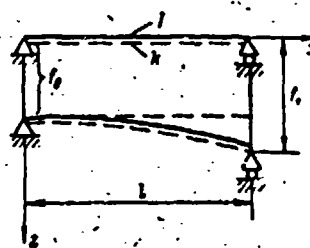


Fig. 74.

89. Determine the rotational angles of the support sections of a freely resting knife-edge beam which is lying on a solid elastic base with rigidity  $k$  when the support deviates by value  $f_0$  and  $f_1$  (Fig. 74).

90. A knife-edge beam with length  $l$  and weight per unit length  $p$  lies on a solid elastic base with rigidity  $k$  (Fig. 75). The left end of the beam is lifted by force  $P = \lambda p l$ , where  $\lambda < 1$ . Compose the equation for determining length  $a$  by which the beam is torn away from the base, considering that the elastic base does not receive tensile forces at this length.

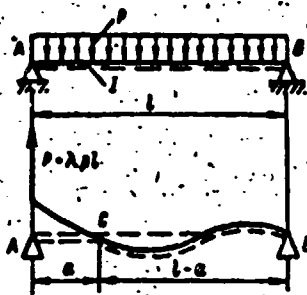


Fig. 75.

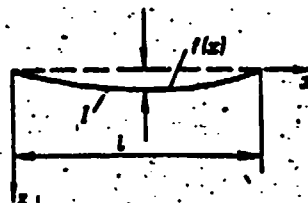


Fig. 76.

91\*. Find the equation for the curve of  $f(x)$  (Fig. 76) by which an elastic base with rigidity  $k = \text{const}$  should be described in order to provide the given bending moment diagram for a beam which lies on this base and which is stressed by a load of intensity  $q(x)$  (the broken line shows the beam's axis in the undeformed state).

92. Find the bending moments  $M_0$  for a freely resting knife-edge beam of length  $l$  which lies on an elastic base with rigidity  $k$  which must be applied to the end sections so that these sections obtain identical positive angles of rotation  $\alpha_0$ . (The moments act in the same direction.)

93. Which forces  $Q/2$  must be applied to the ends of a knife-edge beam (the section's moment of inertia is  $I$ ) which lies freely on an elastic base so that these ends receive the given sagging  $f$ ?

94. Find the change in sagging in the middle of the span, as well as the change in the angle of rotation of the end sections of

a freely resting knife-edge beam lying on an elastic base with rigidity  $k$  and stressed by uniformly distributed load  $q$  after the installation of an elastic support with a pliability coefficient  $A$  in the middle of the span.

95\*. Using the initial parameters method, write out the elastic line of a cantilever knife-edge beam (the moment of inertia of the cross-sectional area is  $I$ ) which lies partially on an elastic base with rigidity  $k$  and which is loaded by force  $P$  (Fig. 77).



Fig. 77.

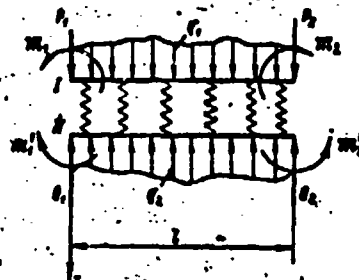


Fig. 78.

96\*. Compose the differential equation for the bending and boundary conditions for a system of two knife-edge beams joined by an elastic base of rigidity  $k$  and stressed by an equilibrium load, end moments and forces (Fig. 78). The solution to this equation can be used to determine the reaction of keel blocks when mooring ships in a floating dock.

97\*. Determine the sagging and bending moment in the middle of a span in the first of two knife-edge beams which are joined by an elastic base with rigidity  $k = \text{const}$  (Fig. 79).

98. A supportless knife-edge beam of length  $l$  which lies on a solid elastic base is stressed by a uniformly distributed load  $q$ . What identical moments  $M_0$  must be applied to the end sections of the beam so that the intensity of the reaction of the elastic

base in the middle of the span of the beam turns out to be equal to zero? What will the intensity of the elastic base's reaction  $r$  be in this case at the ends of the beam (Fig. 80)?

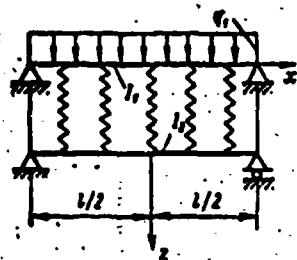


Fig. 79.

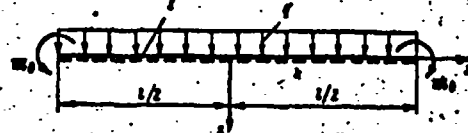


Fig. 80.

99. Determine the moment in the elastic fastening of a knife-edge beam lying on an elastic base, one end ( $x=0$ ) of which is fastened elastically (pliability coefficient is  $\alpha$ ) and the other ( $x=l$ ) of which is completely free, under the effect of a uniformly distributed load of intensity  $q$  and concentrated moment  $M_0$  on the beam, applied in section  $x=l$  (Fig. 81).

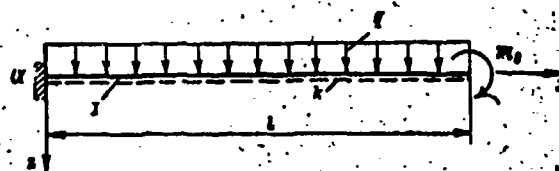


Fig. 81.

100. Determine the angles of rotation of the support sections and the sagging in the middle of the length of a freely resting knife-edge beam which lies on a solid elastic base with rigidity  $k$  with consideration of shear strain and with the assumption that the support sections of the beam received displacement  $f$  (Fig. 82). The rigidity of the beam to shear is  $G\omega$ .

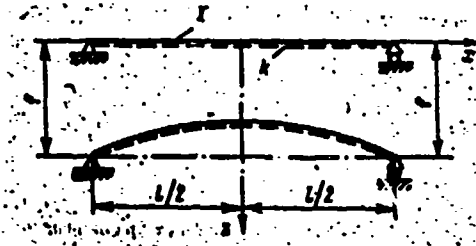


Fig. 82.

101. With consideration of shear deformation, determine the bending moments in the support sections and in the middle of the span of a rigidly fixed knife-edge beam which lies on an elastic base and which is stressed in the middle of the span by concentrated force  $P$ . The beam's length is  $l$ , the moment of inertia of the cross-sectional area is  $I$  and the wall area is  $\omega$ .

102. A knife-edge beam with moment of inertia of the cross-sectional area  $I$  and length  $l$  which lies on a solid elastic base with rigidity  $k$  and which is loaded by a uniformly distributed load with intensity  $q$  has the following arrangement of support sections: a) a beam which rests freely on rigid supports; b) a beam which is rigidly fastened onto elastic supports whose pliability coefficient is  $A$ . Determine the bending moments, the shear forces, the angles of rotation and sagging in the support sections and in the middle of this beam with consideration of shear strain. The wall cross-sectional area is  $\omega$ .

103. A rigidly fixed continuous knife-edge beam which is supported in the span by five equidistant elastic supports with identical rigidity ( $K = \frac{1}{A} = \text{const}$ ) and which are stressed in the support sections by concentrated forces  $P$  (Fig. 83) is replaced by a beam which lies on a solid elastic base with rigidity  $k = \frac{K}{a}$  and which is loaded by a uniformly distributed load of intensity  $q = \frac{P}{a}$ .

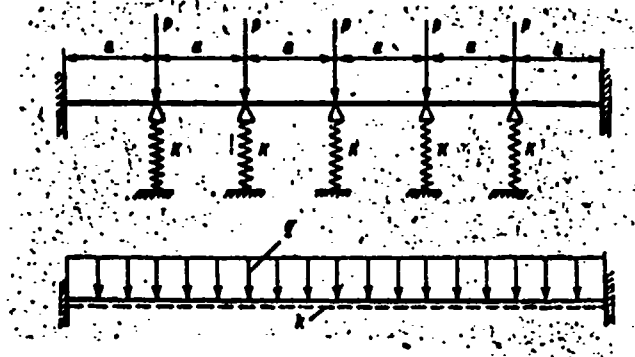


Fig. 83.

The moments of inertia of the cross-sectional area of the beam  $I_0 = 229.9 \cdot 10^3 \text{ cm}^4$ ; the pliability of the elastic supports  $A = 15.32 \frac{\text{cm}^4}{\text{GPa}}$ ; the length of the separate span of the beam  $a = 78 \text{ cm}$ .

Using the solution to problem 102, determine the bending moment and shear force in the fastening ( $M_0, N_0$ ), the bending moment ( $M_1$ ) and the sagging in the middle of the span of the indicated beam on an elastic base.

104. Solve problem 103 with consideration of shear strain. The cross-sectional area of the wall  $\omega = 15.6 \text{ cm}^2$ .

### CHAPTER III.

#### BENDING OF FLAT COVERINGS

##### Brief Theoretical Information

Coverings with one cross connection. The assumption that only the concentrated reactions  $R_1$  perpendicular to the plane of the covering originate in the points of intersection is generally accepted when designing tops consisting of beams in two directions. Furthermore, it is accepted that the external distributed load is taken directly by the beams in the main direction, and the cross connections are only stressed by the reactions of the interaction of the beams in the two directions.

Designing coverings with a large number of supports for the beams in the main direction<sup>1</sup> with identical rigidity and arrangement and one cross connection is reduced to calculating the cross connection as if it were a beam lying on a solid elastic base [see (2.1)]. In this case, the rigidity of the elastic base, the intensity of the load and the argument  $u$  are determined by the formulae:

$$k = \frac{E I_0}{\gamma a l^3}; \quad (3.1)$$

$$q(x) = \frac{\beta(x)}{\gamma} \cdot \frac{Q(x)}{a}; \quad (3.2)$$

$$u = \sqrt[4]{\frac{1}{64} \cdot \frac{1}{\gamma} \cdot \frac{I_0}{l} \left(\frac{L}{l}\right)^3 \frac{L}{a}}; \quad (3.3)$$

<sup>1</sup>It is possible in practice to consider the number of beams in the main direction to be large if there are more than 4-5 of them.

where  $L$  is the length of the covering (the span of the cross connection);  $l$  is the width of the covering (the span of the beams in the main direction);  $a$  is the distance between the beams in the main direction;  $I$  is the moment of inertia of the cross-sectional area of the cross connection;  $I_0$  is the moment of inertia of the cross-sectional area of the beams in the main direction;  $x$  is the coordinate read along the cross connection;  $Q(x)$  is the load on the beam in the main direction which is located at distance  $x$  from the origin of the coordinates;  $\gamma$  is the coefficient of the effect of the concentrated force applied to the beam in the main direction at the point where it intersects the cross connection on sagging at this point;  $\beta(x)$  is the coefficient of the effect of load  $Q(x)$  on sagging of the beam in the main direction in the place where it intersects the cross connection. If the load along the top is constant and coefficient  $\beta = \text{const}$ ,  $q = \text{const}$ .

In order to design any beam in the main direction, it is necessary to find the sagging at the point where it intersects the cross connection. The value of this sagging is found by calculating the cross connection. When determining sagging, the reaction of the interaction of the cross connection with the  $i$ -th beam in the main direction is found:

$$R_i = \frac{EI_0}{\gamma l^3} w_i - \frac{\beta}{\gamma} Q(x), \quad (3.4)$$

where  $w_i$  is the sagging of the  $i$ -th beam in the main direction at the point where it intersects the cross connection, and then the bending moment and shear diagrams are constructed.

If a covering with one cross connection is stressed by concentrated forces, the calculation of the cross connection is reduced to calculating a beam on a solid elastic base which is loaded by concentrated forces. If concentrated force  $P$  is applied directly to the beam in the main direction, the cross connection proves to be stressed in the corresponding section with force

$$P^* = \frac{\beta}{\gamma} P, \quad (3.5)$$

where  $\beta$  is the coefficient of the effect of force  $P$  on sagging at the point of the beam in the main direction's intersection with the cross connection.

The reactions of the cross connection's interaction with the beams in the main direction, to which the external forces are directly applied, are calculated after determining sagging according to formula (3.4).

If the rigidity and the fixing conditions of one of the beams in the main direction are different from the conditions of fixing and the rigidity of the remaining beams, the calculation of the cross connection is reduced to calculating a beam which lies on an elastic base and which is fastened at the point where it intersects a variable beam in the main direction by an elastic support with rigidity

$$K = \left( \frac{\gamma}{\gamma'} n - 1 \right) \frac{EI_0}{\gamma n^2} \quad (3.6)$$

and which is stressed by concentrated force  $P_1$  instead of  $q(x)$ , which is determined by formula (3.2):

$$P_1 = \left( \frac{\beta'}{\gamma'} - \frac{\beta}{\gamma} \right) Q(x), \quad (3.7)$$

where  $\gamma'$  is the coefficient of the effect of the concentrated force applied to the variable beam in the main direction at the point of its intersection with the cross connection on sagging at this point;  $\beta'$  is the coefficient of the effect of the external load acting on the variable beam in the main direction on its sagging at the point of intersection with the cross connection;  $n = I_1/I_0$ ;  $I_1$  is the moment of inertia of the cross-sectional area of the variable beam in the main direction.

Covering with several cross connections. Different methods can be used to calculate coverings with a large number of beams in the main direction and several cross connections (Fig. 84). Below we will consider the method of "main bends" and the method of the "load selection" of cross connections.

**Method of Main Bends.** Assuming that the interaction of beams at joints results only in the same type of reactions perpendicular to the plane of the covering and considering only bending strain of the beams, the system of differential equations which determine the static uncertainty can be written in the form

$$w_i = \beta_i \frac{Q(x)P}{EI_0} - \sum_{j=1}^n \gamma_{ij} \frac{a^2 EI_j w_j^{IV}(x)}{EI_0}, \quad (3.8)$$

where  $w_i$  is the sagging of any beam in the main direction at the point of intersection with the  $i$ -th cross connection;  $\beta_i$  is the coefficient of the effect of load  $Q(x)$  on sagging of the  $i$ -th joint of a beam in the main direction;  $\gamma_{ij}$  is the coefficient of the effect of the reaction in the  $j$ -th cross connection on sagging of the  $i$ -th beam in the main direction;  $a$  is the distance between the beams in the main direction;  $I_j$  is the moment of inertia of the cross sectional area of the  $j$ -th cross connection;  $i=1, \dots, n$  ( $n$  is the number of cross connections)

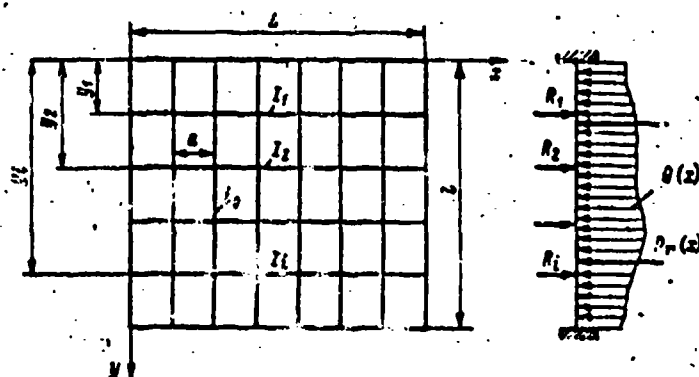


Fig. 84.

System of equations (3.8) can be integrated in the following manner. Let

$$w_i(x) = \frac{1}{T_i} \sum_{m=1}^n v_{im} \rho_m(x), \quad (3.9)$$

where  $I_0$  is a certain constant value which has the dimensionality of the moment of inertia;  $r_{1m}$  are constant coefficients which designate the types of main bends;  $p_m(x)$  are the functions which satisfy the equation for bending of a knife-edge beam which lies on a solid elastic base (main bends):

$$E l_0 p_m^{IV}(x) + k_m p_m(x) = q_m(x) \quad \text{at } m=1, 2 \dots n, \quad (3.10)$$

in which the rigidity coefficients of the elastic base  $k_m$  and the load intensity  $q_m(x)$  must be determined. The following system of algebraic homogeneous equations should be used to calculate the types of main bends  $\nu_{jm}$  and coefficients  $k_m$ :

$$\left. \begin{aligned} & \left( \gamma_{11} - \frac{I_0}{I_1} \lambda_m \right) v_{1m} + \gamma_{12} v_{2m} + \dots + \gamma_{1n} v_{nm} = 0; \\ & \gamma_{21} v_{1m} + \left( \gamma_{22} - \frac{I_0}{I_2} \lambda_m \right) v_{2m} + \dots + \gamma_{2n} v_{nm} = 0; \\ & \dots \\ & \gamma_{n1} v_{1m} + \gamma_{n2} v_{2m} + \dots + \left( \gamma_{nn} - \frac{I_0}{I_n} \lambda_m \right) v_{nm} = 0. \end{aligned} \right\} \quad (3.11)$$

where

$$\lambda_m = \frac{EI_0}{a^2 k_m}$$

System of equations (3.11) has a solution which is different than zero only when its determinant is equal to zero, i. e., if

$$\begin{vmatrix} \left( \gamma_{11} - \frac{I_0}{J_1} \lambda_m \right) & \gamma_{12} & \dots & \gamma_{1n} \\ \gamma_{21} & \left( \gamma_{22} - \frac{I_0}{J_2} \lambda_m \right) & \dots & \gamma_{2n} \\ \dots & \dots & \dots & \dots \\ \gamma_{n1} & \gamma_{n2} & \dots & \left( \gamma_{nn} - \frac{I_0}{J_n} \lambda_m \right) \end{vmatrix} = 0. \quad (3.12)$$

The characteristic numbers  $\lambda_m$  and, consequently, also the rigidity coefficients of the elastic base

$$k_m = \frac{EI_0}{a^3 \lambda_m} \quad (3.13)$$

are determined by equations (3.12).

Since the types of main bends are determined from equations (3.11) with accuracy up to an arbitrary multiplier, all  $v_{mm}$  can be considered to be equal to one ( $v_{mm}=1$ ) and the remaining  $v_{jm}$  can be determined from any  $n-1$  of the equations in system (3.11), substituting all the roots  $\lambda_m$  to equations (3.12) in them in turn. The types of the main bends  $v_{jm}$  satisfy the following condition, called the condition of orthogonality:

$$\sum_{i=1}^n \frac{l_i}{l_i} v_{im} v_{ir} = 0 \quad \text{at} \quad m \neq r. \quad (3.14)$$

System of equations (3.11) and orthogonality conditions (3.14) make it possible to determine the intensity of the load  $q_m(x)$ :

$$q_m(x) = \frac{Q(x)}{a} \cdot \frac{\sum_{i=1}^n \beta_i v_{im}}{\lambda_m \sum_{i=1}^n \frac{l_i}{l_i} v_{im}^2} \quad (3.15)$$

The roots of the characteristic determinant (3.12) for a covering with two cross connections, as well as for a covering with three cross connections and symmetrical load and structure, are determined from the formula

$$\lambda_{1,2} = \frac{A_1}{2} \pm \frac{1}{2} \sqrt{A_1^2 + 4A_2} \quad (3.16)$$

where

$$A_1 = \gamma_{11} \frac{l_1}{l_0} + \gamma_{33} \frac{l_3}{l_0};$$

$$A_2 = (\gamma_{12}^2 - \gamma_{11} \gamma_{22}) \frac{l_2 l_3}{l_0^2}.$$

In this case, the types of main bends are determined by the formula

$$\left. \begin{aligned} v_{11} &= v_{21} = 1; \\ v_{12} &= -\frac{\gamma_{11}}{\gamma_{11} - \frac{I_0}{I_1} \lambda_0}; \\ v_{22} &= -\frac{\gamma_{11} - \frac{I_0}{I_1} \lambda_1}{\gamma_{11}} \end{aligned} \right\} \quad (3.17)$$

When the ends of all the cross beams are fixed identically, the boundary conditions for each of the main bends prove to be the same as for the sagging of the cross beams in question. If the conditions for fixing the support sections are not the same for different cross connections and have the form at  $x=0$

$$\left. \begin{aligned} w_i &= -A_j^0 EI_j \bar{w}_j, \quad \bar{w}_j = \eta_j^0 EI_j \bar{w}_j; \\ \text{at } x=L \\ w_i &= A_j^L EI_j \bar{w}_j, \quad \bar{w}_j = -\eta_j^L EI_j \bar{w}_j \end{aligned} \right\} \quad (3.18)$$

where  $A_j^0$ ;  $A_j^L$  and  $\eta_j^0$ ;  $\eta_j^L$  are the pliability coefficients of the elastic support and fastening of the  $j$ -th cross connection, the boundary conditions in the main bends are not separated, i. e., the main bends will be connected to each other. In this case, using (3.9), condition (3.18) can be written as follows: at  $x=0$

$$\left. \begin{aligned} \frac{I_0}{I_j} \sum_{m=1}^n v_{jm} \rho_m(0) &= -A_j^0 \sum_{m=1}^n N_m^0 v_{jm}; \\ \frac{I_0}{I_j} \sum_{m=1}^n \rho_m'(0) v_{jm} &= \eta_j^0 \sum_{m=1}^n \eta_m^0 v_{jm}; \\ \text{at } x=L \\ \frac{I_0}{I_j} \sum_{m=1}^n v_{jm} \rho_m(L) &= A_j^L \sum_{m=1}^n N_m^L v_{jm}; \\ \frac{I_0}{I_j} \sum_{m=1}^n \rho_m'(L) v_{jm} &= -\eta_j^L \sum_{m=1}^n \eta_m^L v_{jm} \end{aligned} \right\} \quad (3.19)$$

where  $N_m^0$ ,  $N_m^L$  and  $M_m^0$ ,  $M_m^L$  are the shear forces and bending moments which correspond to the m-th main bend:

$$\left. \begin{aligned} N_m^0 &= EI_0 \rho_m'(0); \quad N_m^L = EI_0 \rho_m'(L); \\ M_m^0 &= EI_0 \rho_m(0); \quad M_m^L = EI_0 \rho_m(L). \end{aligned} \right\} \quad (3.20)$$

The types of main bends in condition (3.19) and their first derivatives at  $x=0$  and  $x=L$  can always be expressed by the load which acts on a beam lying on an elastic base and by the value of the shear forces and bending moments in the support sections. In particular, these dependences for a beam which is uniformly stressed by load  $q_m$  and symmetrically fastened at the ends will be the following

$$\left. \begin{aligned} \rho_m(0) &= \rho_m(L) = \frac{2N_m^0}{k_m L \mu_0(u_m)} + \frac{q_m}{k_m} + \\ &+ \frac{2M_m^0}{L^2 k_m \mu_0(u_m)} [\psi_0(u_m) - \psi_1(u_m)]; \\ \rho_m'(0) &= -\rho_m'(L) = \frac{q_m L^3}{24EI_0} \psi_2(u_m) - \\ &- \frac{M_m^0 L}{3EI_0} \left[ \psi_0(u_m) + \frac{\psi_1(u_m)}{2} \right] - \frac{\rho_m(0)}{L} \cdot \frac{8}{3} u_m^4 \psi_2(u_m), \end{aligned} \right\} \quad (3.21)$$

where  $\mu_0(u_m)$ ,  $\psi_0(u_m)$ ,  $\psi_1(u_m)$ , etc. are I. G. Bubnov's functions for beams which lie on an elastic base.

In the case in question, the values of the bending elements in the middle of the length of the beam which correspond to the m-th main bend will be

$$\left. \begin{aligned} \rho_{mcp} &= \frac{q_m L^4}{64EI_0 \mu_0^4} \left[ 1 + \frac{2N_m^0}{q_m L} \cdot \frac{\psi_0(u_m)}{\mu_0(u_m)} + \frac{2M_m^0}{q_m L^3} s(u_m) \right]; \\ M_{mcp} &= M_m^0 \psi_1(u_m) + \frac{N_m^0 L}{4} \cdot \frac{\chi_0(u_m)}{\mu_0(u_m)}, \end{aligned} \right\} \quad (3.22)$$

where

$$s(u_m) = \frac{v_2(u_m) - v_1(u_m)}{\mu_0(u_m)} \varphi_0(u_m) - 4u_m \chi_0(u_m); \quad (3.23)$$

$$u_m = \frac{L}{2} \sqrt{\frac{k_m}{4EI_0}}. \quad (3.24)$$

The bending elements of the cross connections are determined from the formulae

$$\begin{aligned} w_j(x) &= \frac{1}{I_j} \sum_{m=1}^n v_{jm} \rho_m(x); \\ M_j(x) &= EI_j w_j''(x) = \sum_{m=1}^n v_{jm} EI_0 \rho_m''(x) = \sum_{m=1}^n v_{jm} M_m(x); \\ N_j(x) &= EI_j w_j'(x) = \sum_{m=1}^n v_{jm} EI_0 \rho_m'(x) = \sum_{m=1}^n v_{jm} N_m(x), \end{aligned} \quad (3.25)$$

where  $\rho_m, M_m, N_m$  are the bending elements of the beam for the  $m$ -th main bend.

The consideration of shear deformation in the beam walls in both directions has considerable significance in the calculation of bottom coverings with a double bottom. This scheme for calculating coverings with consideration of shear strain can only be used when the ratio of the cross-sectional area of the beam wall  $\omega_1$  to the moment of inertia of all the cross-sectional areas of this beam  $I_1$  for all the cross beams is a constant value at any 1:

$$\frac{\omega_1}{I_1} = \frac{\omega_0}{I_0} = \text{const.} \quad (3.26)$$

In this case, the calculation of the covering is reduced to calculating  $n$  (the number of cross connections) knife-edge beams which lie on an elastic base with rigidity  $k_m$  [formula (3.13)] and which is stressed by distributed load  $q_m$  [formula (3.15)] and axial tensile force  $\frac{EI_0}{G\omega_0} k_m$ .

The equation for determining the main bends in this case is the following

$$EI_0 p_m^{IV} - \frac{EI_0}{G\omega_0} k_m \dot{p}_m + k_m p_m = q_m(x), \quad m = 1, 2, \dots, n, \quad (3.27)$$

where

$$\omega_0 = \frac{I_0}{I_1} \omega_1. \quad (3.28)$$

The summary elastic line of the  $i$ -th cross connection is determined by the equation

$$w_i(x) = w_{1i}(x) + w_{2i}(x), \quad (3.29)$$

where  $w_{1i}(x)$  is the elastic line from bending;  $w_{2i}(x)$  is the elastic line from shear, whereupon

$$w_{2i}(x) = -\frac{EI_1}{G\omega_1} \dot{w}_{1i}(x), \quad (3.30)$$

$$w_{1i}(x) = \frac{I_0}{I_1} \sum_{m=1}^n v_{im} p_m(x). \quad (3.31)$$

The formulae and equations used to determine  $\lambda_m$  and  $v_{im}$  remain the same as in the problem in which shear is not considered. The coefficients of effect should be calculated in this case with consideration of shear in the beam walls in the main direction.

As we have already indicated, the auxiliary functions put into tabular form by Professor N. V. Mattes can be used to determine the design elements of bending of beams on an elastic base with consideration of shear. If condition (3.29) is not satisfied, shear can only be calculated approximately when designing coverings by the method of main bends. For this purpose, it is necessary to use the reduced values  $\bar{I}_1$  of the moments of inertia of the cross connections  $I_1$ , calculated according to formula

$$\bar{I}_1 = \frac{I_1}{\eta}, \quad (3.32)$$

instead of their real values when computing the roots of

characteristic equation (3.12), the types of main bends (3.11) and load intensity, where

$$s_i = 1 + \frac{48}{5-4\pi} \frac{EI_i}{G\omega_i L^3} \quad (3.33)$$

The coefficients of effect  $\gamma_{ij}$  and  $\beta_i$  must be computed with consideration of shear strain in the beams in the main direction. The bending elements in the cross connections will be determined in this case by formulae

$$\left. \begin{aligned} w_i(x) &= w_{ii}(x) s_i = \frac{I_0}{I_i} s_i \sum_{m=1}^n \gamma_{im} p_m(x); \\ w'_{ii}(x) &= \frac{I_0}{I_i} \sum_{m=1}^n \gamma_{im} p'_m(x); \\ M_i(x) &= EI_0 w''_{ii}(x) = \sum_{m=1}^n \gamma_{im} M_m(x); \\ N_i(x) &= EI_0 w'_{ii}(x) = \sum_{m=1}^n \gamma_{im} N_m(x), \end{aligned} \right\} \quad (3.34)$$

where  $p_m(x)$  are the main bends determined by differential equation (3.10);  $p'_m(x)$ ;  $M_m(x)$ ;  $N_m(x)$  are the angle of rotation, the bending moment and the shear in the  $m$ -th main bend, respectively.

Method of "Load Selection" of Cross Connections. The calculation of coverings with several cross connections using the load selection method is based on the concept of the intensity of of the load acting on the  $j$ -th cross connection, as follows

$$r_j(x) = q_j^0(x) - q_j \varphi_j(x), \quad j = 1, 2, \dots, n, \quad (3.35)$$

where  $q_j^0(x)$  is the intensity of the load, calculated with the assumption of the undeformability of the cross connections;  $\varphi_j(x)$  is the selected function whose form is similar to the elastic line of the cross connection  $\left[\varphi_j\left(\frac{L}{2}\right) = 1\right]$ ;  $q_j$  is the unknown

intensity of the load, whose determination requires the use of the equation for sagging of the joint cross sections of the beams in both directions. The system of equations which determines  $q_j$  will be the following

$$\frac{L^4}{EI_{0i}} (\alpha_i q_i^0 + \beta_i q_i) = \frac{a_i^4}{EI_0} \sum_{j=1}^n \gamma_{ij} q_j, \quad (3.36)$$

where  $L$  is the length of the cross connection;  $I_{0i}$  is the mean value of the moment of inertia of the  $i$ -th cross beam;  $q_i^0$  is the mean value of the load in the  $i$ -th cross connection;  $\gamma_{ij}$  is the coefficient of the effect of force  $R_j$  on sagging in the  $i$ -th joint;  $\alpha_i$  and  $\beta_i$  are the coefficients of effect on sagging in the middle of the length of the cross beam under the action of loads with intensity  $q_i^0$  and  $q_i$ , respectively.

When calculating bottom coverings with half-partitions in the diametric plane, it is expedient to take the intensity of the load acting on the vertical keel in the form

$$r_k = q_k^0 + q_k^{(1)} - q_k \varphi_k(x), \quad (3.37)$$

where  $q_k^{(1)}$  is a constant coefficient whose determination requires using the condition of equating the sagging of the beam in the main direction at the point where it intersects the keel in its support section to zero. It is advisable to use the auxiliary functions given in [3, Table I] to simplify the calculations according to the method of load selection.

When designing cross connections using the method of load selection, shear deformation can be considered in the following manner: the beams in the main direction are calculated with consideration of shear; the coefficients of effect  $\gamma_{ij}$ ;  $\beta_i$ ;  $\alpha_i'$  are determined with consideration of shear deformation; shear deformation is considered when determining sagging of cross beams.

## Problems

### Calculating Rectangular Flat Coverings

105. An edge covering consists of a large number of equidistant ribs with a moment of inertia of the cross-sectional area  $I$  and one edge stringer. As Fig. 85 shows, concentrated force  $P$  acts on one of the ribs. Considering that the covering is infinitely long, at what value of the moment of inertia of the cross-sectional area of the edge stringer

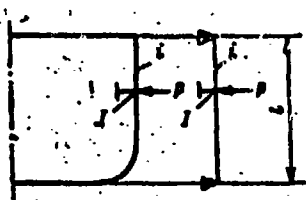


Fig. 85.

$I$  will the sagging of a rib stressed at the point of intersection with the stringer be  $n$  times less than the sagging of the rib without the stringer? The width of the covering is  $b$  and the distance between ribs is  $a$ .

106. Let concentrated forces  $P$  be applied to  $m$  ribs in the covering examined in problem 105 (Fig. 85). Determine how much greater the sagging of the middle of these stressed beams is than the sagging of a stressed beam when one concentrated force is acting on it. When solving the problem, consider that the concentrated force  $P$  is equivalent to a uniformly distributed load of intensity  $q = \frac{P}{a}$  on a section with length  $a$ .

107. A covering of length  $L$  and width  $b$  with one cross connection and a large number of beams in the main direction, separated by distance  $a$ , is stressed by concentrated force  $P$  at the point where the middle beam in the main direction intersects the cross connection. The cross connection (moment of inertia of the cross-sectional area is  $I$ ) is rigidly fastened at the ends, while the beams in the main direction (moment of inertia of the cross-sectional area is  $I$ ) are resting freely. Determine the greatest bending moment in the middle beam in the main direction, if the cross connection passes through the middle of the width of

the covering.

108. A covering with length  $L$  and width  $l=0.8L$  that has one cross connection and a large number of beams in the main direction, the distance between which  $a=0.1L$ , is stressed according to Fig. 86a, b, c, d and e. The boundary conditions for fixing the cross connection (the moment of inertia of the cross-sectional area is  $I$ ) and the beams in the main direction (the moment of inertia

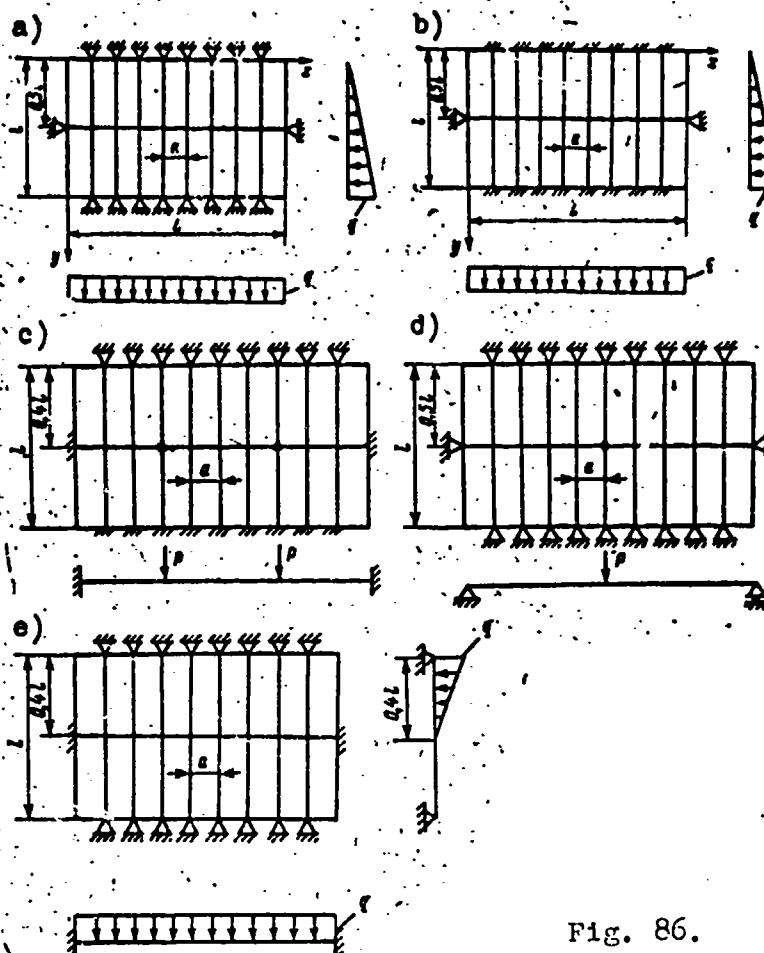


Fig. 86.

of the cross-sectional area  $I=0.1L$ ) are shown in the same figure. Determine the bending moments in the middle beam in the main direction  $m(y)$  and in the cross connection  $M(x)$  in the support sections

and at the point of their intersection, as well as the reaction of the interaction between the cross connection and the middle beam in the main direction.

109\*. A covering with one cross connection and a large number of beams in the main direction, the distance between which is  $a$ , is loaded by a uniformly distributed load of intensity  $q$ . The beams in the main direction with length  $l$  are freely resting at the ends; the moment of inertia of their cross-sectional areas is equal to  $i$ , except for the middle beam, whose moment of inertia is equal to  $m_i$ . The cross connection with length  $L$  (the moment of inertia of the cross-sectional area is  $I$ ) is rigidly fastened at the ends and divides the span of the beams in the main direction in half. Determine the reaction of the interaction of the cross connection with the middle beam in the main direction.

110. An evenly stressed covering consists of a large number of beams in the main direction, the distance between which is  $a$ , and one cross connection. What ratio should there be between the moment of inertia of the cross-sectional area of the cross connection and the moment of inertia of the beam in the main direction of this covering so that the bending moment in the middle beam in the main direction turns out to be four times smaller in the middle of its span than in the absence of the cross connection? The cross connection with length  $L$  is rigidly fixed and divides the width of the covering in half; the beams in the main direction with length  $l$  are resting freely ( $L: l=1$ ;  $L: a=10$ ).

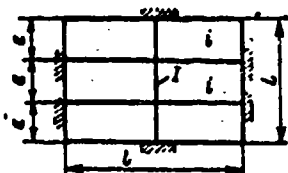


Fig. 87.

111. Establish the dependence of the bending moment in the support sections of the beams in the main direction of a uniformly stressed bottom covering consisting of knife-edge beams (Fig. 87) on the change in the moment of inertia of the cross connection. The cross

connection and the beams in the main direction are rigidly fixed. Only the beams in the main direction are stressed by the external load. The moment of inertia of the cross-sectional area of the beams in the main direction is 1.

112. Determine the value of the moment of inertia  $I$  of the cross connection in problem 111 at which the bending moment in the middle cross section of the beams in the main direction will be equal to zero.

113. Determine the bending moments in the fastening, in the middle of the length of the cross connection and in the middle of the length of the middle beam in the main direction of a covering which consists of ordinary and reinforced beams in the main direction (Fig. 88) with the following data:  $\frac{L}{a} = 12$ ;  $\frac{L}{l} = 1.5$ ;  $l_1 = 1.5l$ ;  $I = 3.0I_1$ . The load along the cross connection does not vary.

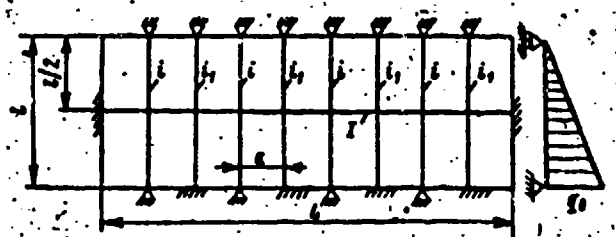
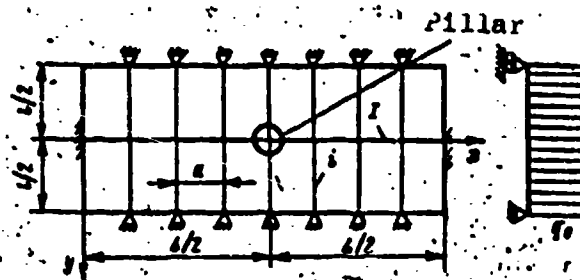


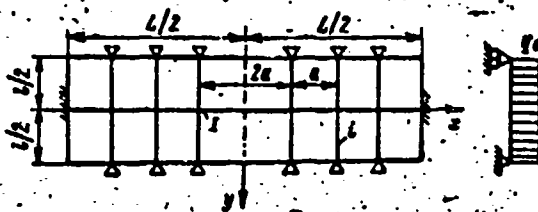
Fig. 88.

114. Determine the bending moments in the cross connection of a covering which is reinforced by a rigid pillar at  $x=0$ ;  $x=\frac{L}{4}$ ;  $x=\frac{L}{2}$  (Fig. 89). Also find the greatest bending moment in the middle beam in the main direction. Given  $\frac{L}{l} = 1.5$ ;  $\frac{L}{a} = 10$ ;  $I = 3I_1$ . the load along the cross connection does not vary.

115. Determine the reaction of the pillar in problem 114 considering its compression, if the cross-sectional area of the pillar is  $F$  and the height of the pillar is  $H$ .



116. Determine the bending moment in the fastening and in the middle of the length of the cross connection of a covering in which one beam in the main direction is absent at  $x=0$  (Fig. 90) with the following data:  $\frac{L}{l} = 1.5$ ;  $\frac{L}{a} = 10$ ;  $\gamma = 3.0$ . The load along the cross connection does not vary.



117. Determine the bending moments in the fastening, the middle of the length of the cross connection and the middle of the length of the middle beams in the main direction of two coverings which are joined by a rigid pillar, as shown in Fig. 91. The load intensity on the lower covering  $q_0 = \text{const}$ ;  $\frac{L}{l} = 1.5$ ;  $\frac{L}{a} = 10$ ;  $\frac{L}{l_1} = 2$ ;  $\frac{L}{l_2} = 6$ ;  $\frac{L}{l} = 3$ .

118\*. Derive the differential equation for the bending of the cross connection of the covering shown in Fig. 92 with the condition that the beams in the main direction are accompanied by torque. The proportionality constant between the angle of twist of the beams in the main direction and the torque is equal to  $c$ ;

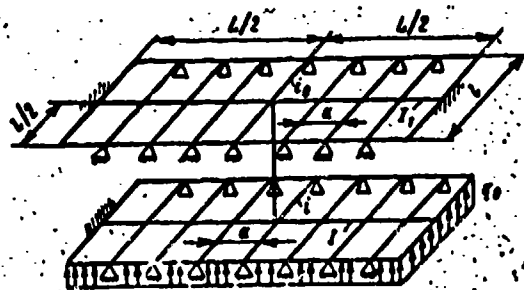


Fig. 91.

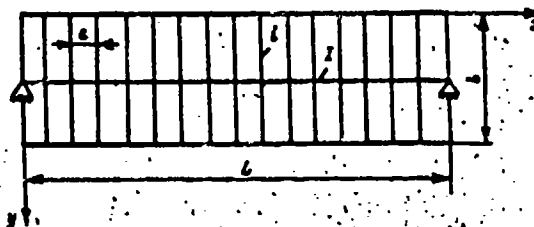


Fig. 92.

there are a rather large number of beams in the main direction. Obtain the general expression for the elastic line of a cross connection in the form of a trigonometric series, considering that the end sections of the connection are resting freely on rigid supports.

119\*. Calculate a bottom covering which is uniformly stressed by a load of intensity  $q$  (Fig. 93). The cross connections, stringers and vertical stabilizer are rigidly fixed, while the beams in the main direction (floors) are freely supported. Make the calculation using the method of main bends without consideration of shear in the walls (using appendix VIII) and with consideration of shear in the walls. Given:  $\frac{l}{L} = 0.8$ ;  $\frac{a}{L} = 0.1$ ;  $\frac{c}{l} = 0.25$ ;  $\frac{l_c}{l_n} = 0.8$ ;

$$\frac{l_\phi}{l_n} = 0.5; \frac{\omega_\phi}{\omega_n} = 0.8; \frac{l_n}{\omega_n l^2} = 0.015.$$

121. Compose the equation for determining the sagging and moments in the support sections for the first and second main bends in a covering with two different cross connections with the condition that one of the cross connections is rigidly fixed and the other is completely free in the end sections (the load is evenly distributed).

Given:  $\frac{L}{a} = 29$ ;  $\frac{L}{l} = 1.38$ ;  $\frac{c}{l} = 0.25$ ;  $\frac{l}{l_c} = \frac{1}{7}$ ;  $\frac{l_n}{l_c} = 1.2$ .

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directionless pressure (Fig. 95). The upper ends of the ribs are freely supported, while the lower ends are rigidly fixed. The stringers are rigidly fixed at the ends. Carry out the calculation with and without consideration of shear.

Given:  $\frac{l}{L} = 0.8$ ;  $\frac{l_1}{a_1 L^3} = 0.015$ ;  $\frac{c_1}{L} = 0.18$ ;  $\frac{l_2}{a_2 L^3} = 0.01$ ;  $\frac{c_2}{L} = 0.36$ ;  $\frac{l_3}{l_1} = 0.5$ ;

$\frac{a}{L} = \frac{1}{15}$ ;  $\frac{l}{l_1} = \frac{1}{3}$ ;  $\frac{l}{a L^3} = 0.005$ .

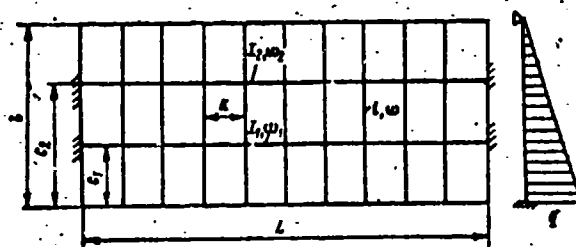


Fig. 95.

124. Using the method of main bends, determine the bending moments in the support sections and in the middle of the span of the cross connections of a uniformly stressed bottom covering with lengthwise half-partitions in the center-line plane. The basic dimensions of the covering are given in Fig. 96. The vertical stabilizer and the stringer are rigidly fastened; the floors are freely supported. Given:  $\frac{l_c}{l_n} = \frac{1}{2}$ ;  $l_1 = l_n$ ;  $l_2 = 0.5 l_n$ .

125. Solve problem 124 using the method of the "load selection" of cross connections. The intensity of the reaction of the elastic base is given in the following form:  
for the stringer

$$q_c = \frac{1}{2} q_1 \left( 1 - \cos \frac{2\pi x}{L} \right) + \frac{1}{2} q_2 \left( 1 - \cos \frac{4\pi x}{L} \right);$$

for the vertical stabilizer

$$q_n = \frac{1}{2} q_1^n \left( 1 - \cos \frac{2n\pi x}{L_n} \right) - q_2^n.$$

Equate the following points when composing the basic equations for sagging of cross connections and beams in the main direction:  $x_1 = 5.5a$ ;  $x_2 = 3a$ .

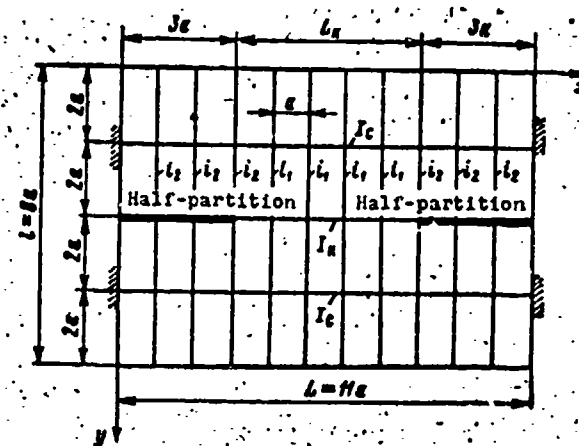


Fig. 96.

126. Determine the bending moments in the calculated cross sections of the vertical stabilizer and stringer of the covering considered in problem 119 with the condition that the covering is stressed by one concentrated force in the middle of the stabilizer. Make the calculation using the method of main bends in two versions: a) without consideration of shear; b) with consideration of shear.

127. Determine the sagging (in cm) and bending moments in the middle of the span and also the bending moments in the support sections of a vertical stabilizer and a stringer (in t.m) for a uniformly stressed bottom covering (Fig. 97). The stabilizer and the stringer are rigidly fastened to the partitions and the floors are resting freely on the edges. Make the calculation with and

without consideration of shear strain by means of the method of main bends.

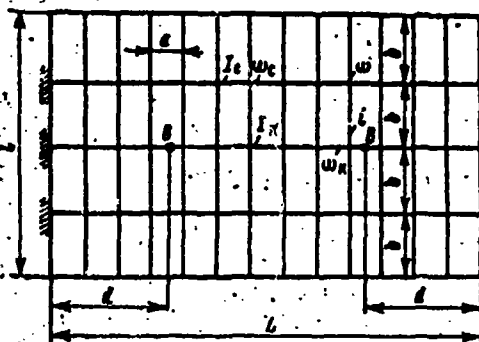


Fig. 97.

Given:  $L_1 = 22.1$  m;  $l = 16.1$  m;  $a = 0.762$  m;  $b = 3.86$  m;  $I_k = 370$  cm<sup>2</sup> m<sup>2</sup>;  $I_c = 310$  cm<sup>2</sup> m<sup>2</sup>;  $i = 75$  cm<sup>2</sup> m<sup>2</sup>; the areas of the beam walls  $\omega_k = 140.5$  cm<sup>2</sup>;  $\omega_c = 49$  cm<sup>2</sup>;  $\omega = 65$  cm<sup>2</sup>; the calculated pressure  $q$  (t/m<sup>2</sup>).

128. Solve problem 127 by the method of "load selection" of cross connections (without consideration of shear).

129. Solve problem 127 by the "load selection" method, considering that the keel is rigidly fixed and the stringers are elastically fastened (the support pair coefficient  $\alpha = 0.65$ ).

130. Calculate the covering in problem 127, assuming that it is stressed by two concentrated forces  $P$  which are applied to the keel at points B at a distance of  $d = 571$  cm from the partitions.

131. Determine the connection between the reactions and the pliability coefficient  $A$  of elastic supports installed at points B of the covering described in problem 127, assuming that the covering is stressed by a uniform load and the ends of the cross connections are rigidly fastened. Use the main bends method without consideration of shear.

## CHAPTER IV.

### GENERAL THEOREMS OF STRUCTURAL MECHANICS

#### Brief Theoretical Information

1. The strain energy of an elastic system (beams) is designated as the work which must be expended to transform this system from the undeformed state to the deformed state. In the simplest strain situations, the strain energy of a beam (generally, a curvilinear beam) is expressed by the following dependences:

potential energy of bending

$$V_{\text{bnd}} = \frac{1}{2} \int \frac{M^2 ds}{EI} = \frac{1}{2} \int EI (w_1')^2 ds, \quad (4.1)$$

where the integrals are computed along the entire length  $s$  of the shaft's axis;

potential shear strain energy

$$V_{\text{shear}} = \frac{1}{2} \int \frac{N^2 ds}{G\omega} = \frac{1}{2} \int G\omega (w_2')^2 ds, \quad (4.2)$$

potential dilatational-compressive energy

$$V_{\text{dilat}} = \frac{1}{2} \int \frac{T^2 ds}{EF}, \quad (4.3)$$

potential twisting energy

$$V_{\text{twist}} = \frac{1}{2} \int \frac{M_{\text{tp}}^2 ds}{C}, \quad (4.4)$$

where  $M_{xp}$  is the torque;  $C$  is the rigidity of the beam to twisting.

Since the previously considered situations of beam strain are independent, the beam strain energy is equal to the sum of the potential energies of the separate types of strain:

$$V = V_{bx} + V_{b\omega} + V_{b\theta} + V_{bp}$$

or

$$V = \frac{1}{2} \int \left( \frac{M^2}{EI} + \frac{N^2}{G\omega} + \frac{T^2}{EF} + \frac{M_{xp}^2}{C} \right) ds. \quad (4.5)$$

We will note that the formulae above are only correct for linearly deformable systems, i.e., for systems in which the generalized forces and generalized displacements<sup>1</sup> are related by linear dependences with constant coefficients. Furthermore, these formulae can only be used to compute the potential energy of shafts with little curvature, since the situations of strain in question cannot be considered to be independent for shafts with large curvature.

If a beam is rigidly fixed and supported on an elastic support or it is on a solid elastic base, the strain energy of the elastic fastening, the elastic support and the elastic base can be computed using the following formulae:

the strain energy of an elastic fastening arranged in section

$S=C$

$$V_{\text{fnp},s} = \frac{1}{2} 2l M^2(c) = \frac{1}{2 2l} [w_1'(c)]^2, \quad (4.6)$$

the strain energy of an elastic support arranged in section

$S=C_1$

<sup>1</sup>The concepts of generalized displacement and generalized force are given in the theoretical mechanics course.

$$V_{\text{res. en}} = \frac{1}{2} AR^2(c_1) = \frac{1}{2} \frac{(w_1(c_1) + w_2(c_1))^2}{\lambda}, \quad (4.7)$$

where  $R(c_1)$  is the reaction of the elastic support;

the potential strain energy of the elastic base

$$V_{\text{res. en}} = \frac{1}{2} \int_0^{s_1} k (w_1 + w_2)^2 dx, \quad (4.8)$$

where  $s_1$  is the length of the elastic base.

The following theorems are correct for linearly deformable systems: the Castigliano theorem, the theorem of the least work and the theorem of the reciprocity of displacements.

The Castigliano Theorem. The partial derivative of the potential energy for a generalized force is equal to the generalized displacement corresponding to this force:

$$\frac{\partial V}{\partial Q_k} = q_k, \quad (4.9)$$

where  $Q_k$  is the generalized force;  $q_k$  is the generalized displacement which corresponds to generalized force  $Q_k$ . The generalized displacement will be positive if the generalized force of this displacement generates positive work.

The Castigliano theorem is used to determine the generalized displacements of static determinable elastic systems. In order to do this, it is necessary to write the expression of the system's potential energy [e.g., according to formula (4.5)] as a function of the assigned external load and the corresponding unknown generalized displacements of the generalized forces. If the load of the elastic system in question does not include a generalized force which corresponds to the unknown generalized displacement  $q_p$ , it is necessary to introduce fictitious force  $Q_p$  and to set this force equal to zero after finding the generalized displacement, i.e.,

$$q_p = \left( \frac{\partial V}{\partial Q_p} \right)_{Q_p=0} \quad (4.10)$$

It is more convenient to differentiate by the sign of the integral when determining the generalized displacements using the Castigliano theorem than to compose the expression for the potential energy in form (4.5) and then differentiate it. The formula for determining the generalized displacement obtained from this operation is called the Maxwell-Mohr formula:

$$q_k = \int \left( \frac{M}{EI} \frac{\partial M}{\partial Q_k} + \frac{N}{G\omega} \frac{\partial N}{\partial Q_k} + \frac{T}{EF} \frac{\partial T}{\partial Q_k} + \frac{M_{sp}}{C} \frac{\partial M_{sp}}{\partial Q_k} \right) ds. \quad (4.11)$$

The Theorem of the Least Work. The partial derivative of the system's potential strain energy with respect to the excess unknown is equal to zero

$$\frac{\partial V}{\partial R} = 0, \quad (4.12)$$

where R is the excess unknown. The theorem of the least work is used to disclose the static indeterminance of systems. The static indeterminable reactions imposed on the system of connections (the support reactions and moments) or the forces of the interaction of parts of the system on the cross section can be used as the excess unknowns. Differentiation by the sign of the integral is recommended during the use of the theorem of the least work in practice.

The Theorem of the Reciprocity of Displacements. When two systems of loads are acting on an elastic body, the work of the forces of the first state on the displacements in the second state corresponding to them is equal to the work of the forces of the second state on the displacements in the first state corresponding to them.

It follows from the theorem of the reciprocity of displacements that if two separate generalized forces act on an elastic system, the generalized displacement corresponding to the first generalized force, which is caused by the action of the second force, is equal to the generalized displacement corresponding to the second generalized force, which is caused by the action of the first force.

Since the generalized coordinates for linearly deformable systems can be expressed by the generalized forces using the equation

$$q_i = \sum_{k=1}^n \alpha_{ik} Q_k, \quad i = 1, 2, \dots, n, \quad (4.13)$$

on the basis of the beginning of the reciprocity of displacements, coefficients  $\alpha_{ik}$  must satisfy the condition of reciprocity, i.e.,

$$\alpha_{ik} = \alpha_{ki}. \quad (4.14)$$

Coefficients  $\alpha_{ik}$  are called the coefficients of effect.

2. The Ritz method, based on the principle of possible displacements, can be used to study the equilibrium state of an elastic system (linearly and nonlinearly deformable). This method can be stated in the following manner for systems under the action of forces with a potential: the partial derivative of the total energy of the system with respect to the generalized displacement is equal to zero, i.e.,

$$\frac{\partial \mathcal{E}}{\partial q_k} = 0, \quad (4.15)$$

where  $\mathcal{E} = U - V$  is the total energy of the system;  $U$  is the power function of the external load (the work of the external load);  $V$  is the system's potential energy.

The Ritz method is used in particular in bending problems to find the elastic line of beams and in stability problems to

determine the Euler load of beams. In both cases, the elastic line of the beam will have the form of a series

$$w(x) = \sum_{n=1}^{\infty} q_n \varphi_n(x), \quad (4.16)$$

where  $q_n$  are the unknown generalized displacements;  $\varphi_n(x)$  are the functions selected in a like fashion (the fundamental functions). The fundamental functions in (4.16) must satisfy the kinematic boundary conditions, i.e., the conditions regarding sagging and the angles of rotation. Here the fundamental functions need not satisfy the excess kinematic conditions, or else we will obtain the solution for a beam with other boundary conditions.

Having selected the system of fundamental functions, it is necessary to compute the power function for the external load  $U$  and the strain energy  $V$ . When a distributed load acts on the shaft and concentrated force  $P$  and moment  $M_0$ , respectively, are applied to cross sections  $x=c$  and  $x=d$ , the power function will take on the form:

$$U = \int_0^l q(x) w(x) dx + Pw(x=c) + M_0 w'(x=d), \quad (4.17)$$

where  $l$  is the length of the shaft. When longitudinal compressive forces  $T(x)$  are acting on the beam, the power function is calculated according to formula

$$U = \frac{1}{2} \int_0^l T(x) [w'(x)]^2 dx. \quad (4.18)$$

Formulae (4.17) and (4.18) are correct for a linearly deformable system. The values of the power function components are positive if the corresponding forces generate positive work at the displacements. The strain energy  $V$  for linearly deformable systems is determined from the formulae given in p. 1 of this section.

The following system of equations can be written after computing  $U$  and  $V$  according to (4.15):

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\* Both bending and shear indices of beam sagging can be represented in form (4.16).

$$\frac{\partial(U-V)}{\partial q_k} = 0; \quad k = 1; 2; 3; \dots; m, \quad (4.19)$$

where  $m$  is the number of terms in series (4.16), which must be limited when solving specific problems.

Since potential energy  $V$  for linearly deformable systems is the quadratic function of generalized displacements of  $q_n$  and power function  $U$  is the linear function of  $q_n$  in bending problems (the term which is the quadratic function of  $q_n$  is added in complex bending problems), system of equations (4.19) is a system of heterogeneous linear algebraic equations with respect to  $q_n$ , from which they must be determined. The strain energy  $V$  and work of the external forces  $U$  for linearly deformable systems in stability problems are the quadratic functions of the generalized coordinates, resulting in the fact that system of equations (4.19) is a system of homogeneous algebraic equations with respect to the generalized coordinates of  $q_n$ . This system allows solutions other than zero only when its determinant is equal to zero. The unknown value of the Euler force is the smallest root of the characteristic equation.

## Problems

### Bending of Straight Beams

132. Using the Castigliano theorem, determine the sagging from bending in cross section A of a knife-edge beam which is resting freely on two rigid supports for the following load situations:  
 a) the beam is stressed by two concentrated forces (Fig. 98);  
 b) the beam is stressed by two concentrated forces and a distributed load (Fig. 99); c) the beam is stressed by two concentrated forces and a support moment (Fig. 100); d) the beam is stressed by two support moments  $M_0$  (Fig. 101). Section A is at a distance of  $l/4$  from the left support.

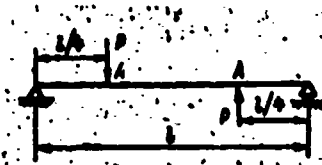


Fig. 98.

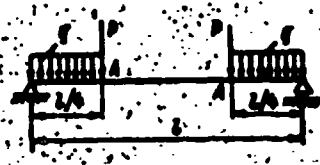


Fig. 99.

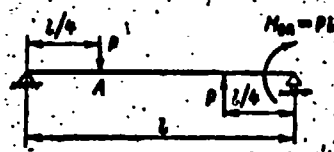


Fig. 100.

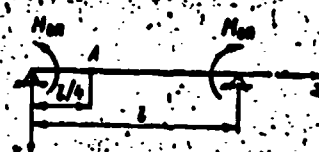


Fig. 101.

133. Using the Castigliano theorem, determine the settling of an elastic support which is freely supported by a knife-edge beam (Fig. 102) with consideration of only displacement due to bending.

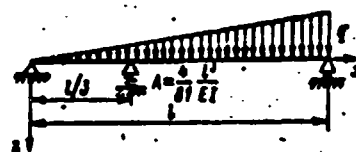


Fig. 102.

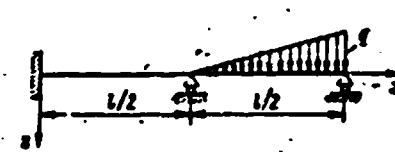


Fig. 103.

134. Using the Castigliano theorem, find the angle of rotation of the cross section which coincides with the middle support of a knife-edge beam (Fig. 103). The moment of inertia of the cross-sectional area of the beam is  $I$ .

135. Determine the moment which must be applied to the right support section of a knife-edge beam (Fig. 104) so that the angle of rotation of this cross section is equal to zero.

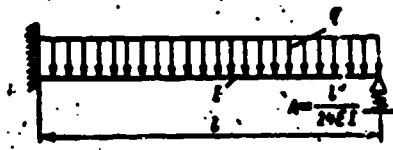


Fig. 104.

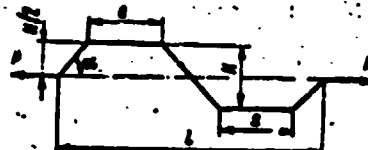


Fig. 105.

136. Determine how much greater the elongation of a stretched broken beam (Fig. 105) is than that of a straight beam which is stretched by the same forces  $q$ . The moment of inertia and the cross-sectional area of the beams are equal to  $I$  and  $F$ , respectively

137. Using the Castigliano theorem, determine the sagging of a cross section under force  $P$  of a knife-edge beam which is resting freely with consideration of shear. The moment of inertia of the cross-sectional area and the area of the wall of the beam are equal to  $I$  and  $\omega$ , respectively (Fig. 106).

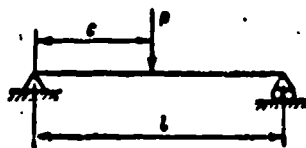


Fig. 106.



Fig. 107.

138. Using the theorem of the least work, find the moment in a rigid beam (Fig. 107) with consideration of shear strain. The

moment of inertia of the cross-sectional area and the wall area are equal to  $I$  and  $\omega$ , respectively.

139. Using the Castigliano theorem, find the sagging of the free end of a cantilever beam stressed by a concentrated force at the end, with consideration of shear strain. The moment of inertia of the cross-sectional area and the wall area of the beam are equal to  $I$  and  $\omega$ , respectively.

140\*. Under the action of concentrated force  $P$ , which is applied on the end of a cantilever knife-edge beam, the elastic line is equal to

$$w(x) = -\frac{PP^0}{3EI} \left[ \frac{x^3}{6} \left( \frac{3}{2} - \frac{x}{2l} \right) \right].$$

Determine the sagging of the end of the cantilever under the action of a load of intensity  $q$ , as shown in Fig. 108.

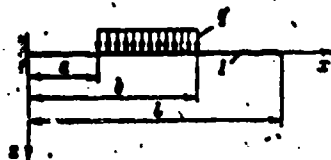


Fig. 108.

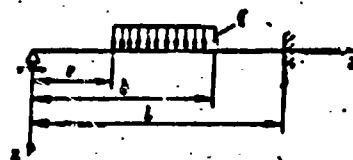


Fig. 109.

141. Given the elastic line of a beam with a constant cross section which rests freely on rigid supports under the effect of moment  $M_0$  which is applied to the right support ( $x=l$ ):

$$w(x) = -\frac{M_0 l^2}{6EI} \left( \frac{x}{l} - \frac{x^3}{l^3} \right).$$

Find the moment in the fixing for the beam in Fig. 109. The moment of inertia of the cross-sectional area of the beam is  $I$ .

142. A knife-edge beam which is elastically fixed at  $x=l$  (pliability coefficient of the elastic fixing is  $\alpha$ ) and which rests freely on an elastic support at  $x=0$  (pliability coefficient

of the elastic support is A) is stressed in section  $x=c$  by concentrated force P. Determine the settling of the elastic support using the beginning of the least work.

143. Using the theorem of the least work, how can the equation for determining the static indeterminance of a beam (Fig. 110) be composed if the following are taken as the unknown: a) support reaction R on the middle support (the reaction is directed downward); b) bending moment  $M_{on}$  in the middle support?

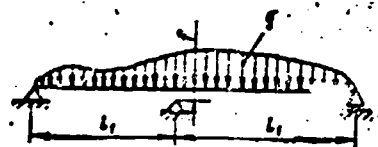


Fig. 110.

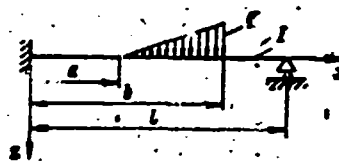


Fig. 111.

144. Given an elastic line of a cantilever beam with a constant cross section under the action of concentrated force P which is applied to the end ( $x=l$ ):

$$w(x) = \frac{P^2}{3EI} \left[ \frac{x^3}{6} \left( \frac{3}{2} - \frac{x}{2l} \right) \right].$$

Find the reaction of the right beam support (Fig. 111) using the theorem of the reciprocity of displacements.

145. Find the line of the effect of a single load on the bending moment at point B of a two-span beam (Fig. 112) which is fastened elastically on the left by a rigid support with the following data:  $l_1 = l_0$ ;  $l_2 = 1.5l_0$ ;  $l_3 = 1.6l_0$ ;  $l_4 = 3l_0$ ;  $2I = \frac{I_0}{3}$ .

146. Find the line of the effect of a single load on the shear force at point B of a beam (Fig. 113) which is rigidly fastened on the left end. One intermediate support is elastic and

the other is rigid. Given:  $I_1 = I_2 = I_0$ ;  $I_3 = 2I_0$ ;  $I_4 = I_5 = I_0$ ;  $c = 0.5l_0$ ;  $A = \frac{8}{10E7_0}$ .

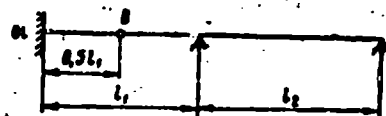


Fig. 112.

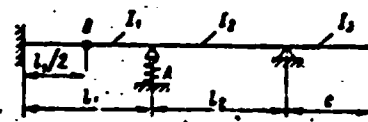


Fig. 113.

147. Using the Ritz method, determine the elastic line of a knife-edge beam (Fig. 114). The fundamental function is taken as  $\psi(x) = x^2$ .

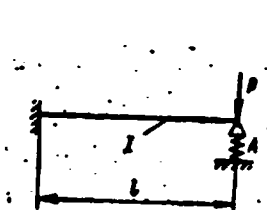


Fig. 114.

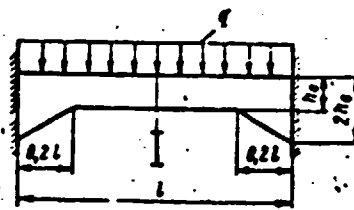


Fig. 115.

148. Select the fundamental function in the form of the polynomial for a knife-edge beam which is rigidly fixed on the left end and freely supported on the right. Find the elastic line of this beam, leaving two terms of the series in the expansion of sagging. The beam is stressed by a uniformly distributed load of intensity  $q$ .

149. Find the static indeterminance of a rigidly fastened nonprismatic T-beam (Fig. 115) which is stressed by a uniformly distributed load of intensity  $q$  and determine its sagging in the middle of the span. Calculate the integrals in the computation process in the tabular form, dividing the length of the beam into

ten equal sections. The wall height in the middle portion is  $h$  and at the fastening -  $2h$ . The width of the bands is constant along the entire beam and is equal to  $0.5h$ ; the band and beam thicknesses are identical and are equal to  $t$ .

### Bending of Curvilinear Beams and Frames

150. A curvilinear frame (Fig. 116) is stressed by horizontal force  $P$ . Determine the moment in the fixing.

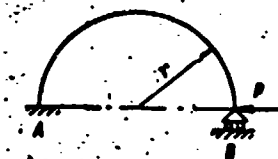


Fig. 116.

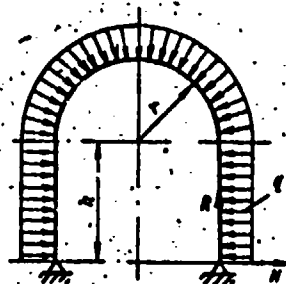


Fig. 117.

151. Disregarding potential shear and dilatational (compressive) energy, find the static indeterminance of curvilinear assemblies composed of shafts with a constant cross section (Fig. 117, 118). Also construct the bending moment and shear diagrams. Take  $h = r$ ;  $\rho = 2r$ .

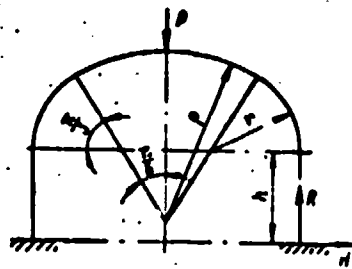


Fig. 118.

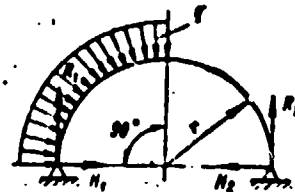


Fig. 119.

152. Disregarding potential shear and dilatational (compressive) energy, find the static indeterminance of a curvilinear knife-edge beam (Fig. 119) whose moment of inertia of the cross-sectional area is  $I$  using the theorem of the least work.

153. Considering only potential bending energy, determine the vertical displacement of cross section A of a curvilinear knife-edge beam loaded in cross section B (Fig. 120) by concentrated moment  $M_0$  using the Castigliano theorem. The moment of inertia of the cross-sectional area of the beam is  $I$ .

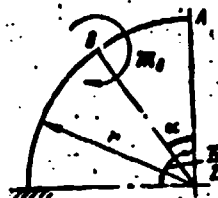


Fig. 120.

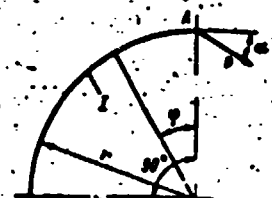


Fig. 121.

154\*. Determine the angle at which force  $P$  should be directed to the horizontal axis of a curvilinear knife-edge beam (Fig. 121) so that the displacement of the point where the force is applied due to the bending of the beam alone only takes place in the direction of this force.

155. Determine the bending moment in any cross section of a round ring which is stressed: a) by two concentrated forces (Fig. 122); b) by concentrated force  $P$  and tangential forces balancing it which are distributed on the perimeter of the ring and which vary according to the law  $q = \frac{P}{\pi R} \sin \theta$  (Fig. 123).

156. Determine the bending moments, shear forces and axial forces in sections 1, 2, 3 and also the force in the partition under the effect of a uniform pressure of intensity  $q$  on the ring

for an oval ring composed of round shafts with a constant moment of inertia and which is fixed by clasp action (Fig. 124).

Take  $R = 2r$ .

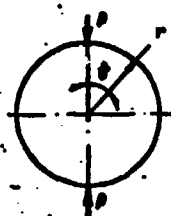


Fig. 122.

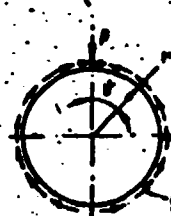


Fig. 123.

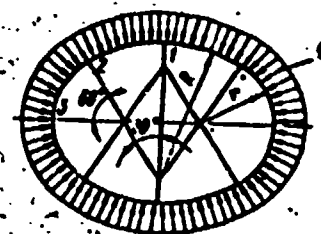


Fig. 124.

157. Determine the bending moments, shear forces and axial forces in sections 1, 2 and 3 of an oval ring composed of round shafts, the moments of inertia of whose cross-sectional areas are equal to  $I_1$  and  $I_2$  (Fig. 125), which is stressed by a uniformly distributed load of intensity  $q$ . Take  $R = 2r$ ;  $I_2 = 2I_1$ .

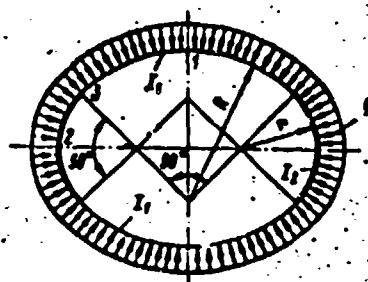


Fig. 125.

#### Bending of Beams on an Elastic Base

158\*. A semi-infinite beam which lies on an elastic base with rigidity  $k = \text{const}$  is stressed by a load (Fig. 126, 127). Determine the bending moment in the cross section of the beam which coincides with the origin of the coordinates (rigid fixing on a sliding support) if we know that the elastic line of the

semi-infinite beam is determined by the expression

$$w_{\text{ш}}(x) = \frac{M_0}{2a^3 EI} (\cos ax - \sin ax) e^{-ax}$$

under the effect of moment  $M_0$  applied at the origin of the coordinates.

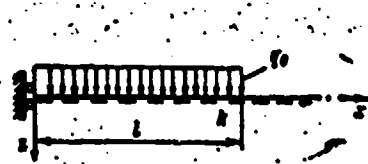


Fig. 126.

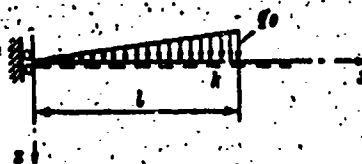


Fig. 127.

159. Determine the angle of rotation of the cross section which coincides with the origin of the coordinates for a semi-infinite beam which lies on an elastic base with rigidity  $k$  and which is stressed in cross section  $x = l$  by concentrated moment  $M_0$  (Fig. 128) if we know that the angles of rotation of the cross section of the beam in question are determined by the expression

$$w'(x) = \frac{M_0}{aEI} e^{-ax} \cos ax$$

under the effect of the moment applied at the origin of the coordinates.

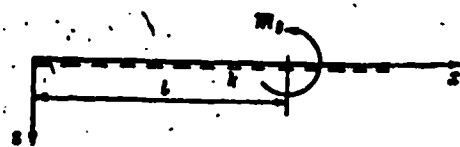


Fig. 128.



Fig. 129.

160. Determine the bending index at the origin of the coordinates of a semi-infinite knife-edge beam which lies on an elastic base with rigidity  $k$  and which is stressed according to Fig.

129 if we know that the elastic line of the beam is equal to

$$w(x) = \frac{P}{2EIa^2} e^{-ax} \cos ax.$$

under the effect of concentrated force  $P$  applied at the origin of the coordinates.

161. Using the theorem of the least work, compose the equation for calculating a rigidly fixed beam stressed by a uniformly distributed load of intensity  $q$  which lies on a solid elastic base of rigidity  $k$ . Take  $M_0$  as the unknown support moment at  $x = 0$  and  $M_1$  at  $x = l$  and the intensity of the reaction of the elastic base  $r = \sum r_n \sin \frac{n\pi x}{l}$ .

162. How does the solution to problem 161 change if shear in the beam wall is also taken into consideration? The wall's cross-sectional area is  $\omega$ .

163. Using the theorem of the least work, compose the equation for calculating rectangular coverings with one cross connection which is rigidly fastened to partitions. The cross connection's length is  $L$ ; the moment of inertia of the cross-sectional area is  $I$ . The width of the covering is  $l$  and the moment of inertia of the cross-sectional area of the beams in the main direction is  $i$ . Take the intensity of the reactions of the interaction of the cross connection and the beams in the main direction as the unknowns

$$r = r_0 + \sum r_n \sin \frac{n\pi x}{L},$$

where  $r_0$  is the intensity of the reaction in the case of an absolutely rigid cross connection. Consider that there are large number of beams in the main direction.

164. Obtain the expression for the elastic line of a knife-edge beam (Fig. 130) using the Ritz method. Take

$$w(x) = a_1 x + a_2 x^2; \quad \Delta l = \frac{l}{6EI}; \quad k = 45 \frac{EI}{l^3}.$$

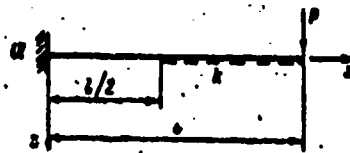


Fig. 130.

### Complex Bending and Shaft Stability

165. Using the Ritz method, determine the elastic line of the knife-edge beam in Fig. 131. Look for the elastic line in the form  $w(x) = a_1 x^2$ .

166. Using the Ritz method, determine the elastic line of a knife-edge beam which rests freely at the ends and is stressed by constant longitudinal dilational forces  $T$  for the following situations of stressing the beam with a transverse load: a) concentrated force  $P$  acts in the span of the beam at  $x = c$ ; b) the beam is stressed by a uniformly distributed load of intensity  $q$ . The beam's length is  $l$ ; the moment of inertia of the cross-sectional area is  $I$ . Search for the beam's elastic line in the form

$$w(x) = \sum_{n=1}^{\infty} a_n \sin \frac{n\pi x}{l}.$$

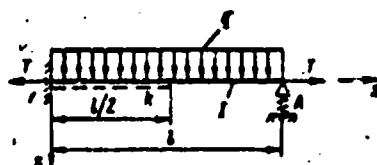


Fig. 131.

167. Using the Ritz method, determine the elastic line of a knife-edge beam which is rigidly fastened at the ends and stressed by constant longitudinal compressive forces  $T$  and a uniformly distributed load of intensity  $q$ . The beam's length is  $l$  and the moment of inertia of the cross-sectional

area is  $I$ . Search for the beam's elastic line in the form

$$w(x) = \frac{a_1}{2} \left( 1 - \cos \frac{2\pi x}{l} \right).$$

Find the bending index of the beam in the

middle of its length ( $x = l/2$ ) at  $T = 0.6 \frac{4\pi^2 EI}{l^2}$  and compare with the precise solution.

168. Using the Ritz method, determine the value of the Euler force for a freely resting knife-edge beam which is compressed by a constant force and which is resting on an elastic base with piecewise-constant rigidity (Fig. 132), taking the form of the stability loss in the form  $w(x) = A \sin \frac{\pi x}{l}$ . The moment of inertia of the beam's cross-sectional area is  $I$  and the length  $-l$ .

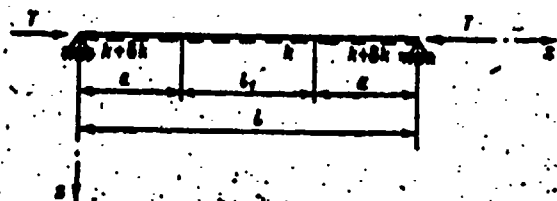


Fig. 132.

169. Using the Ritz method, determine the Euler force of a freely supported beam whose cross-sectional area moment of inertia varies according to the law (the origin of the coordinates is taken in the support section of the beam)  $I = I_0 \sin \frac{\pi x}{l}$ , where  $l$  is the length of the beam. The form of the stability loss is taken in the form:

$$\begin{aligned} \text{a) } w(x) &= a_1 \sin \frac{\pi x}{l}; \\ \text{b) } w(x) &= a_1 \sin \frac{\pi x}{l} + a_2 \sin \frac{3\pi x}{l}. \end{aligned}$$

170. Using the energy method, obtain the value of the critical rigidity  $K_{sp}$  of the elastic supports of the knife-edge beam in Fig. 133.

171. Using the energy method, obtain the value of the Euler force for a freely resting knife-edge beam, part of the length of which is supported by an elastic base with a rigidity coefficient

k (Fig. 134). Take the form of the stability loss of the shaft as follows:  $w(x) = \sin \frac{\pi x}{L}$ .



Fig. 133.

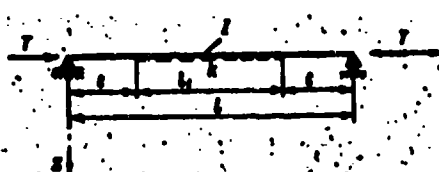


Fig. 134.

172. Using the Ritz method, determine the Euler force for a freely supported beam with a staggered cross section which is compressed by a stepped-variable force (Fig. 135). Take the form of the stability loss as follows:  $w(x) = \sin \frac{\pi x}{L}$ , where  $L = 2l + l_0$ . How does the expression for the Euler force change if we consider the effect of deviation from Hooke's law on stability?

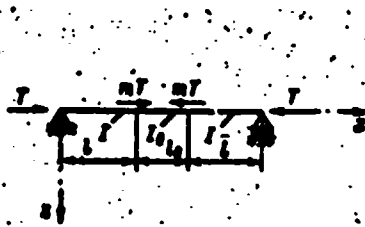


Fig. 135.



Fig. 136.

173\*. Using the Ritz method, determine the Euler force for the assembly shown in Fig. 136, assuming that the shafts in the frame bend one half-cycle of a sine wave. Consider the effect of deviation from Hooke's law on stability.

174. Using the Ritz method, find the Euler force of a

centrally compressed knife-edge beam which is freely supported on the ends with consideration of shear strain. The beam's length is  $l$ , the moment of inertia of the cross-sectional area and the area of the wall section are  $I$  and  $\omega$ , respectively. Express the potential bending energy of the beam by bending moments and the potential shear energy by shear forces.

175. Using the Ritz method, determine the Euler force of a knife-edge beam (Fig. 137) which is under the effect of a compressive load distributed according to the law  $P(x) = P_0 \frac{x}{l}$ . The form of the stability loss is taken in the form  $w(x) = a(x - \frac{2}{l}x^2 + \frac{1}{l^2}x^3)$ .

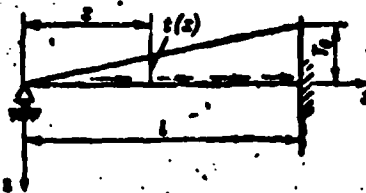


Fig. 137.

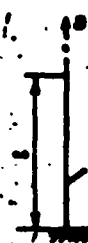


Fig. 138.

176. Determine the Euler load of a knife-edge column (Fig. 138) under the effect of its own weight, using the Ritz method. The weight of a unit length of the column is  $q$ . The form of the stability loss is taken as follows:  $w(x) = a_1 x^2 + a_2 x^3$ .

177. Using the Ritz method, determine the Euler force on a nonprismatic beam which is rigidly fixed at  $x = 0$  and which is completely free at  $x = l$ . Take the form of the stability loss as follows:  $w(x) = ax^2$ . The moment of inertia of the beam's cross-sectional area varies according to the law  $I(x) = I_0(1 - \frac{x}{l})$ .

178. Using the Ritz method, determine the Euler force for a knife-edge beam which is elastically fastened at the ends (flexibility coefficient  $\alpha = \frac{l}{6ET}$ ) and which is fastened in the

middle of the span by an elastic support. The pliability coefficient of the elastic support  $A = \frac{P}{EI}$ . Take the form of the stability loss in the form  $w(x) = a_1 \sin \frac{\pi x}{l} + a_2 \sin \frac{3\pi x}{l}$ .

179. Solve problem 178 for the situation in which one end of the beam is freely supported and the other is completely free. The form of the stability loss is taken as follows:  $w(x) = a_1 x + a_2 x^2$ . The pliability coefficient of the elastic support  $A = \frac{P}{EI}$ .

## CHAPTER V.

### COMPLEX BENDING, SHAFT STABILITY AND THE SIMPLEST SHAFT SYSTEMS

#### Brief Theoretical Information

1. The differential equation for bending of a beam which supports transverse  $q(x)$  and longitudinal dilational  $T(x)$  loads (complex bending) and which has initial sagging  $w_0(x)$  is written in the form

$$[EI(x)w''(x)]' - [T(x)w'(x)]' = q(x) + [T(x)w_0'(x)]'. \quad (5.1)$$

The differential equation for complex bending of beams with a constant cross section which are under the action of a constant longitudinal force ( $I = \text{const}$ ,  $T = \text{const}$ ) is written in the form

$$EIw''''(x) - Tw''(x) = q''(x). \quad (5.2)$$

where

$$q''(x) = q(x) + Tw_0'(x).$$

The integral of equation (5.2) can be written as follows

$$w(x) = A_0 + A_1 k'x + A_2 \text{ch } k'x + A_3 \text{sh } k'x + w_{0,p}(x),$$

where  $A_i$  are the random constants;  $w_{0,p}$  is the partial solution;

$$k' = \sqrt{\frac{T}{EI}}.$$

If the intensity of the transverse load varies according to the linear law  $q(x) = q_0 + mx$ , in the absence of initial sagging the partial solution has the following appearance:

$$w_{0,p}(x) = -\frac{q_0 x^2}{2T} - \frac{mx^3}{6T}. \quad (5.4)$$

If the axial force is compressive, we should set

$$T = -T^*, \quad (5.5)$$

in all the preceding formulae, where  $T^*$  is the absolute value of the axial compressive force. Then the integral of equation (5.2) will be as follows

$$w(x) = B_0 + E k^* x + B_1 \cos k^* x + B_2 \sin k^* x + w_{ep}(x), \quad (5.6)$$

where

$$k^* = \sqrt{\frac{T^*}{EI}},$$

and the partial solution to (5.4) is transformed to the form

$$w_{ep}(x) = \frac{q_0 x^3}{24 T^*} + \frac{m x^3}{6 T^*}. \quad (5.7)$$

It is necessary to write out two boundary conditions each in order to determine the integration constants at each end of the beam. In general, when the ends of the beam are fixed elastically onto elastic supports, the boundary conditions have the form (the origin of the coordinates is taken on the left end of the beam) at  $x = 0$

$$\left. \begin{aligned} w &= A_1 [T (w_0 + w') - EI w'']; \\ w' &= \alpha_1 EI w''; \\ w &= A_2 [-T (w_0 + w') + EI w'']; \\ w' &= -\alpha_1 EI w''. \end{aligned} \right\} \quad (5.8)$$

where  $A_1$  and  $A_2$  are the pliability coefficients of the left and right elastic supports;  $\alpha_1$  and  $\alpha_2$  are the pliability coefficients of the elastic fastening in the same supports.

The boundary conditions for all of the simpler situations can be obtained from (5.8).  $T$  must be replaced by  $-T^*$  for compressive forces in conditions (5.8).

Remember that the principle of the addition of the action of forces can be used in problems of complex bending only when each of the terms corresponds to the same value of the longitudinal force.

2. These dependences can be used to solve problems of the complex bending of flexible plates of a cylindrical surface. For this purpose, the cylindrical rigidity of the beam of band  $D = \frac{Eh^3}{12(1-\nu^2)}$  should be used in these expressions instead of beam rigidity  $EI$ , while we mean the pressure and axial force which act on a beam-band of identical width by  $q$  and  $T$ . If the beam-band subjected to complex bending is located on a solid elastic base (the problem of bending of a round cylindrical shell under unidirectional load), the differential equation for bending, its integral and the auxiliary functions which make it possible to determine the bending elements of the beam-band can be obtained by using dependence [3, Table II].

3. The Euler force of single-span shafts can be determined using integral (5.6) at  $w_{n,p}(x) = 0$ . Making this integral adhere to the boundary conditions, we will have a system of homogeneous algebraic equations with regard to the integration constants. Equating the determinant of this system to zero, we should find the smallest value  $k^* = k^*_{\min}$ . Then the value of the Euler force will be equal to

$$T_e = EI k_{\min}^2. \quad (5.9)$$

Formula (5.9) can only be used when the Euler stress  $\sigma_e = T/F$ , where  $F$  is the cross-sectional area of the shaft, does not exceed the proportionality limit. If this condition is not satisfied, the actual stresses at which loss of the beam's stability occurs (critical stress  $\sigma_{kp}$ ) are determined from formula  $\sigma_{kp} = \sigma_e \eta$ , where  $\eta$  is the coefficient which accounts for the effect of the deviation from Hooke's law on stability (determined depending on the ratio of the Euler stress to the yield point of the shaft material [1, Fig. 159]).

When studying the stability of shafts which lie on a solid elastic base, the differential equation

$$EI\left(1 - \frac{T^*}{G\omega}\right) w_1^{IV}(x) + \left(T^* - k \frac{EI}{G\omega}\right) w_1''(x) + kw_1(x) = 0, \quad (5.10)$$

can be used with consideration of shear deformation, where  $w_1(x)$  is the sagging from bending;  $G\omega$  is the rigidity of the shaft to shear;  $k$  is the rigidity coefficient of the elastic base.

The form of the integral in equation (5.10) is generally very complex; however, for a beam which is freely supported at the ends it is possible to set

$$w_1 = a_n \sin \frac{n\pi x}{l}, \quad (5.11)$$

where  $n$  is the number of half-waves of stability ss.

If shear strain is not taken into consideration ( $G\omega = \infty$ ), the integral of equation (5.10) is written in the form

$$w(x) = C_1 \cos px + C_2 \sin px + C_3 \cos ix + C_4 \sin ix, \quad (5.12)$$

where

$$\left. \begin{aligned} p &= \alpha \sqrt{2(\beta_0 + \sqrt{\beta_0^2 - 1})}; \quad i = \alpha \sqrt{2(\beta_0 - \sqrt{\beta_0^2 - 1})}; \\ \beta_0 &= \left| \frac{T}{2\sqrt{EI/k}} \right|; \quad \alpha = \sqrt{\frac{k}{4EI}}. \end{aligned} \right\} \quad (5.13)$$

Making (5.12) adhere to the boundary conditions, we will have a system of homogeneous equations. The Euler force should be determined as indicated above.

If there is no elastic base, but shear strain is taken into consideration, the value of  $T$ , is determined according to formula (5.9), substituting  $\frac{k_{\min}^2}{1 + k_{\min}^2 \frac{EI}{G\omega}}$  in it for  $k_{\min}^2$ .

4. The equations for the continuity of the angular deformations on the supports with consideration of axial forces  $T$  and moments  $M_1$  (the type of scheme with five or three moments) should be composed when studying the stability of continuous shafts which are supported on elastic (or rigid) supports. In addition to these equations, the dependences which relate the displacements of these supports  $w_1$  to their reactions  $R_1$  are composed for beams on elastic supports. In order for the system of linear homogeneous algebraic equations thus obtained to allow the solutions  $M_1 \neq 0$ ;  $w_1 \neq 0$  (or  $M_1 \neq 0$  for shafts on rigid supports), it is necessary to equate the determinant of this system to zero, the least root of which will be determined by the system's Euler load. If a shaft of length  $l$  is freely supported on two edge supports and the intermediate elastic supports have identical rigidity and are the same distance from each other, the relationship between the required rigidity of these supports  $K$  and the compressive load  $T$  is established by the formula

$$K = \eta \frac{\pi^2 E I}{a^3} \chi_j(\lambda)_{\min}, \quad (5.14)$$

where  $\eta = \sigma_{sp}/\sigma_c$ ;  $E I$  is the beam's rigidity;  $a = \frac{l}{s+1}$  is the distance between supports;  $s$  is the number of elastic supports;  $\lambda = \frac{\sigma_{sp}}{\sigma_c}$  is a parameter;  $\sigma_c^0 = \frac{\pi^2 E I}{a^2 F}$ ;  $T = \sigma_{sp} F$ ;  $F$  is the cross-sectional area of the beam;  $\chi_j(\lambda)$  is the function of parameter  $\lambda$  and numbers  $n$  and  $j$  ( $1 \leq j \leq n$ ), determined according to Table 1 of Appendix V.

The critical rigidity of the elastic supports can be determined from formula (5.14), assuming that  $\lambda = 1$  and  $\eta = \sigma_c^0/\sigma_c$ , where  $\sigma_c^0$  is the critical stress which corresponds to  $\lambda = 1$ . The increase in the rigidity of the supports above the critical value does not result in an increase in the compressive load supported by the shaft.

5. The problem of the stability of a flat rectangular deck covering consisting of knife-edge (generally, different) compressed lengthwise beams which rest freely at the ends on rigid supports

and which are equidistant from identical transverse beams which are arbitrarily fastened at the ends is solved on the basis of the assumptions given in p. 4 of this section and by means of the problem of the vibrations of a weightless transverse beam which supports concentrated masses at the points where it intersects the lengthwise beams.

Using the solution to these problems, it is possible to obtain the equations for the stability of various types of coverings. The basic design formulae are given below, in which the following notations are used:

- $l$  - the length of the covering;
- $L$  - the width of the covering;
- $l_1 = \alpha L$  - the middle portion of the width of the covering which is not reinforced by lengthwise beams;
- $a = \frac{l}{s+1}$  - the distance between beams;
- $s$  - the number of beams;
- $b$  - the distance between ordinary lengthwise beams;
- $b_1$  - the distance between reinforced lengthwise beams (carlings);
- $F$  - the cross-sectional area of ordinary lengthwise beams with given bands of width  $b$ ;
- $F_1$  - the cross-sectional area of a reinforced lengthwise beam (carling) with the given band of width  $b_1$ ;
- $i$  - the moment of inertia of the cross-sectional area of an ordinary lengthwise beam with a given band;
- $i_1$  - the moment of inertia of the cross-sectional area of a reinforced lengthwise beam with the given band;
- $\sigma_{kp}$  - the critical stress of the covering;
- $\eta$  - the coefficient which accounts for the effect of deviation from Hooke's law on stability;

$$\sigma_s^0 = \frac{\pi^2 E i}{a^2 F};$$

$$\sigma_{s_1}^0 = \frac{\pi^2 E i_1}{a^2 F_1};$$

$$\lambda = \frac{\sigma_{kp}}{\eta \sigma_s^0};$$

$$\lambda_1 = \frac{\sigma_{kp}}{\eta \sigma_{s_1}^0};$$

$\chi_j(\lambda)$ ;  $\chi_j(\lambda_1)$  - the functions determined from Table 1 of Appendix V depending on parameter  $\lambda$  (or  $\lambda_1$ ) and numbers  $n$  and  $j$ , where  $j$  is the number of half-waves of the form of stability loss ( $1 \leq j \leq n$ ).

The covering consists of a large number of identical equidistant lengthwise beams and identical equidistant beams which are elastically fixed at the ends (see Fig. 159). The moment of inertia of the beams which provides the covering with the assigned compressive stress is determined by the expression

$$I = \left(\frac{\pi}{\mu_1}\right)^4 \left(\frac{L}{a}\right)^3 \frac{L}{b} \eta \chi_j(\lambda)_{\max}. \quad (5.15)$$

where coefficient  $\left(\frac{\pi}{\mu_1}\right)^4$  for coverings which are reinforced by lengthwise beams along their entire width is determined according to Table 2 of Appendix V depending on the conventional support pair coefficients of the beam fastenings:

$$\mu_1 = \frac{1}{1 + \frac{2\eta_1 EI}{L}}; \quad \mu_2 = \frac{1}{1 + \frac{2\eta_2 EI}{L}};$$

$\eta_1$  and  $\eta_2$  are the pliability coefficients of elastic fixings of the beams onto supports.

$\left(\frac{\pi}{\mu_1}\right)^4$  are determined according to Table 3 of Appendix V for coverings in which the lengthwise assembly is missing in the middle section at length  $l_1 = \frac{1}{2}L$  (see Fig. 160) and the beams are fixed symmetrically, depending on the conventional support pair coefficient of the beams' fastening  $\mu = \frac{1}{1 + \frac{2\eta EI}{L}}$  and number  $n = \frac{L}{l}$ .

The values of  $\chi_j(\lambda)$  are determined from Table 1 in Appendix V. A specific value of the moment of inertia of the transverse beams, called the critical value, exists for this type of covering. The further increase in the moment of inertia does not result in an increase in the compressive stress which the covering can support. The critical moment of inertia can be computed according to formula (5.15) at  $\lambda = 1$  and  $\eta = \frac{\sigma_{cr}^0}{\sigma_0}$ , where  $\sigma_{cr}^0$  is the critical stress which corresponds to  $\sigma_0$ .

The covering consists of a large number of equidistant lengthwise beams, some of which are reinforced, and equidistant identical beams which are elastically fastened at the ends (see Fig. 161). The moment of inertia of the beams which provides the covering with the assigned compressive stress

$$I = \left(\frac{\pi}{\mu_j}\right)^4 \left(\frac{L}{a}\right)^3 \frac{L}{b} \eta \left(1 + \frac{b}{b_1} \beta\right) i_{\lambda_j}(\lambda), \quad (5.16)$$

where  $\beta = \frac{i_{\lambda_j}(\lambda_1)}{i_{\lambda_j}(\lambda)} - 1$ , and  $\left(\frac{\pi}{\mu_j}\right)^4$  is determined from Table 2 in Appendix V. At assigned  $\lambda$  and  $\lambda_1$ , functions  $\chi_j(\lambda)$  and  $\chi_j(\lambda_1)$  which are entered in formula (5.16) must be determined for the value of  $j$  at which the moment of inertia of the transverse beams is the greatest.

The covering consists of a large number of identical equidistant lengthwise beams and identical equidistant beams which are resting freely on a carling and are elastically fixed to the edge (Fig. 162). The moment of inertia of the beams which provides the covering with the assigned compressive stress is determined from the formula

$$I = \left(\frac{\pi}{\mu_j}\right)^4 \left(\frac{L}{a}\right)^3 \frac{L}{b} \eta \chi_j(\lambda). \quad (5.17)$$

Parameter  $\mu_j$  is the root of equation

$$F(\mu_j; \kappa) = \frac{b}{L} \left[ 1 - \frac{1}{\mu_j} \frac{\chi_j(\lambda_1)}{\chi_j(\lambda)} \right],$$

and the numerical values of function  $F(\mu_j, \kappa)$  are given in Table 17 of textbook [1]. The value of  $j$  is selected so that the moment of inertia of the beams is the greatest.

Sometimes it is necessary to determine the critical stress according to the given dimensions of the covering when studying the stability of these coverings. This problem is solved graphically, subsequently assigning the values of  $\sigma_{\kappa p}$ , for otherwise it is impossible to determine the value of  $\eta$ .

## Problems

### Using the Differential Equation for Complex Bending

180. Find the equation for the elastic line of a cantilever knife-edge beam which is stressed on the free end by transverse force  $P$  and which is compressed by axial forces  $T$ . Determine the bending moment in the fastening at  $T = \frac{EI}{\rho}$ .

181. Find the equation for the elastic line of a beam (Fig. 139) and the value of sagging in the middle of its span.

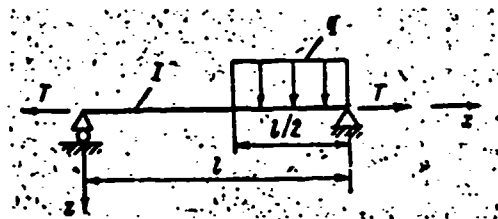


Fig. 139.

182. A rigidly fastened knife-edge beam is compressed by forces  $T$ . Determine the reactions and the support moments if the support cross section of the beam ( $x = 0$ ) rotated by angle  $w'(0)=1$ .

183. A rigidly fastened knife-edge beam is compressed by forces  $T$ . Determine the reactions and the support moments if the right end of the beam ( $x = l$ ) received sagging  $w(l)=1$ .

184. Find the static indeterminance of a continuous knife-edge beam which is compressed by force  $T = 0.4 \frac{\pi^2 EI}{l^2}$ , where  $I$  is the moment of inertia of the cross-sectional area of the beam,  $3l$  is the length of the beam (Fig. 140) and  $A = \frac{l^3}{8EI}$ .

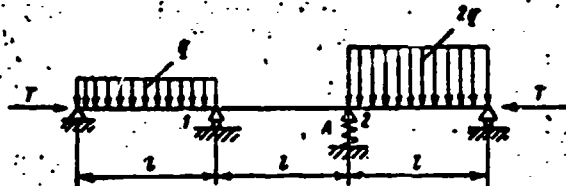


Fig. 140.

185. Find the value of the dilatational axial force which acts on a beam (Fig. 141) if the sagging of the elastic support increases two times in its absence;  $\Delta = \frac{l^3}{24ET}$ .



Fig. 141.



Fig. 142.

186. Obtain the approximate equation for the elastic line of a freely supported beam-band with an initial sagging  $w_0(x) = \frac{c_1}{2} \left(1 - \cos \frac{2\pi x}{l}\right)$ , which is stressed by a uniform load of intensity  $q$  and compressive forces  $T$ . Solve the problem by the Bubnov-Galerkin method, leaving one term of the series in the expansion for sagging.

187. Determine the maximum summary stress in the meridional and cross sections of the covering of a round cylindrical shell which is reinforced by ribs and stressed by an omnidirectional uniform pressure with the following initial data: shell radius of 2.75 m, shell thickness of 0.02 m, distance between ribs (spacing) of 0.60 m, cross-sectional area of the rib of  $3.5 \cdot 10^{-3} \text{ m}^2$ , pressure of 39 kg/cm<sup>2</sup>, shell material normal elasticity modulus of  $2 \cdot 10^6 \text{ kg/cm}^2$ .

188. Obtain the equation for the stability of the shaft depicted in Fig. 142. Determine the Euler force at  $A = \frac{P}{12EI}$ :

189. Find the Euler force for the structure in Fig. 143.

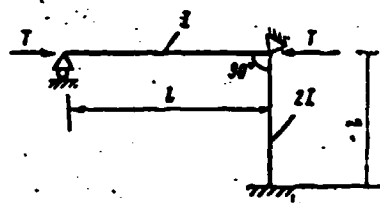


Fig. 143.

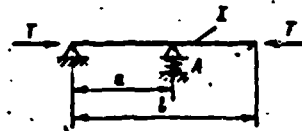


Fig. 144.

190. Study the stability of a knife-edge beam with an intermediate elastic support (Fig. 144) by integrating the differential equation of equilibrium. Determine the Euler force at  $a = \frac{l}{2}$ :  $A = \frac{P}{8\pi^2 EI}$ .

191. Determine the Euler force and critical rigidity of the supports of the knife-edge beam shown in Fig. 145.

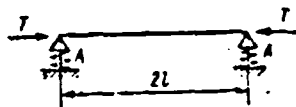


Fig. 145.

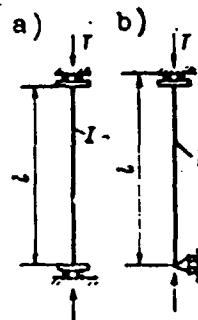


Fig. 146.

192. Determine the Euler force T for a knife-edge beam which is rigidly fastened to sliding elastic supports (Fig. 146a) without

resorting to integrating the differential equation for neutral equilibrium. We know that the Euler force of the beam shown in Fig. 146b  $P_E = \frac{\pi^2 EI}{4l^2}$ .

193. Determine the Euler force of the knife-edge beam in Fig. 147 by integrating the differential equation of equilibrium.

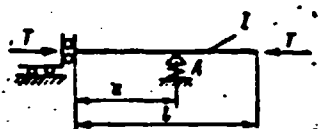


Fig. 147.

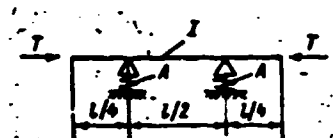


Fig. 148.

194. Using the solutions to problems 190 and 193, determine the Euler force for the knife-edge beam shown in Fig. 148;  $A = \frac{P}{4\pi^2 EI}$ .

195. Compose the equation for the stability of a beam with alternating rigidity which is freely supported on rigid immobile and mobile supports and which is stressed by an axial compressive force applied at the point where the beam's rigidity changes (Fig. 149). Determine the Euler force at  $a = \frac{l}{2}$  and  $\epsilon = \frac{l}{2}$ .

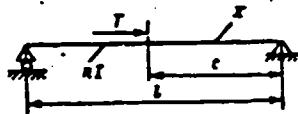


Fig. 149.

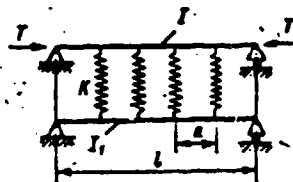


Fig. 150.

196\*. A freely supported compressed beam is connected to a freely supported beam with the same length  $l$  and moment of inertia  $I_1$  (Fig. 150) by equidistant frequently placed partitions with rigidity  $K$ . Determine the Euler force of the compressed beam.

197. Find the Euler load for the structures shown in Figures 151 and 152.

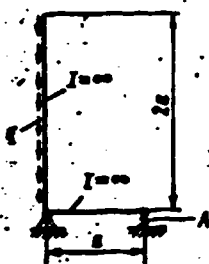


Fig. 151.

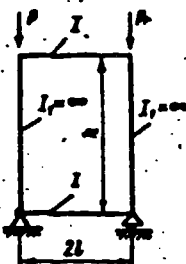


Fig. 152.

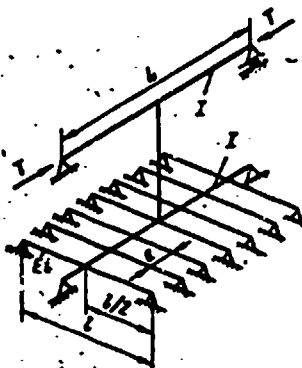


Fig. 153.

198. Find the Euler force for a knife-edge beam, the left end of which is supported on a rigid support and the right end of which is rigidly fastened, with consideration of shear strain. The beam's length is  $l$ , the moment of inertia of the cross-sectional area is  $I$  and the wall area is  $\omega = \frac{EI}{G\alpha^2} = 0.026$ .

199. Determine the Euler force of a knife-edge shaft which lies on a solid elastic base with rigidity  $k = 300 \frac{EI}{l^3}$  and is freely supported on the left end and rigidly fixed on the right.

200. Obtain the expression for the Euler force of a freely supported knife-edge beam which lies on a solid elastic base of rigidity  $k$  with consideration of shear strain. Determine  $T_1$  and the number of waves of stability loss  $n$  for the situation when

$$\lambda = 10 \frac{\pi^2 EI}{P}; \frac{EI}{Gad} = 0.025.$$

How does the result change if shear is not taken into consideration?

201. A freely supported compressed beam (Fig. 153) is supported through an incompressible pillar in the middle of the span on a cross connection of the same length and rigidity. The cross connection is connected to the beams in the main direction. The latter, with length  $l$ , are freely supported and are installed at distance  $a$  from each other. What moment of inertia  $I$  must the beams in the main direction have so that the Euler force of the compressed shaft is  $P_0 = \frac{4\pi^2 EI}{L^2}$ ?

202. Determine the moment of inertia  $I$  of the cross-sectional area that cantilever rigidly fixed knife-edge beams in the main direction which support a compressed freely supported knife-edge cross beam must have so that the Euler force of the latter is equal to  $P_0 = \frac{3\pi^2 EI}{L^2}$ . The length of the cantilever beams is  $l$ ; the distance between them  $a \ll L$  (Fig. 154).

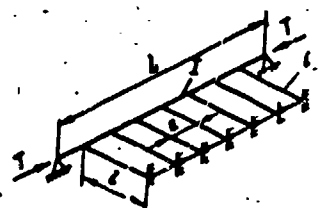


Fig. 154.

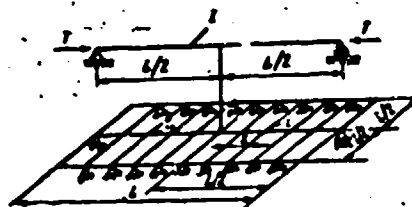


Fig. 155.

203. Solve problem 202, assuming that the beams in the main direction are freely supported at the ends and that the cross beam passes through the middle of the width of the covering. The moment of inertia of the cross connection is equal to  $I$ . Assume

$$L:l = 1; L:a = 10.$$

204. Study the stability of a knife-edge beam which is fixed in the middle of the length by a rigid pillar which is connected to the covering (Fig. 155). Find the critical moment of inertia of the cross connection  $I_{sp}$  for the ratios  $\frac{L}{l} = 1.5$ ;  $\frac{L}{a} = 10$ ;  $l_1 = 3l$  i.e., that moment of inertia whose further increase does not result in an increase in the Euler force of the compressed beam.

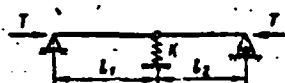


Fig. 156.

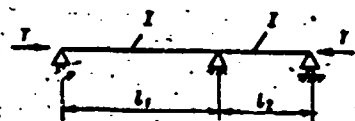


Fig. 157.

205. Determine the Euler force of a freely resting shaft which consists of two shafts with infinitely high rigidity joined together by a hinge and which is reinforced at the place where the hinge is located by an elastic support of rigidity  $K$  (Fig. 156).

206. Find the Euler force for a continuous knife-edge beam (Fig. 157) with the condition that  $l_2 = 1.4l_1$ .

207. Solve the preceding problem with the assumption that the left end of the beam is rigidly fixed.

208. Determine the Euler force of a knife-edge beam (Fig. 158), if  $A = \frac{2}{3} \frac{EI}{l_1}$  and  $l_2 = 2l_1$ .

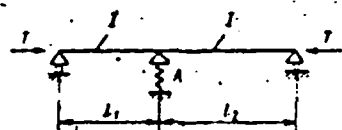


Fig. 158.

209. How many intermediate equidistant identical elastic supports must a shaft with length  $l$  and rigidity  $EI$  which is freely supported at the ends have so that its Euler force increases seven times? The rigidity of each support is equal to  $K = 4.1 \frac{\pi^2 EI}{l^3}$ .

The effect of deviation from Hooke's law on stability is not taken into consideration.

210. Determine the critical stress of a knife-edge shaft which is freely supported at the ends which is on three equidistant identical elastic supports of rigidity  $K = 610 \text{ kg/cm}$  with the following initial data:  $l = 8 \text{ m}$ ;  $F = 4 \cdot 10^{-3} \text{ m}^2$ ;  $I = 8 \cdot 10^{-6} \text{ m}^4$ ;  $\sigma_r = 3000 \text{ kg/cm}^2$ ;  $E = 2 \cdot 10^6 \text{ kg/cm}^2$ .

211. Find the Euler force of a beam which is freely supported at the ends and reinforced by five equidistant identical supports with rigidity  $K = 8.2 \frac{\pi^2 EI}{l^2}$ . The beam's rigidity is  $EI$  and its length is  $l$ .

How does the result change if the rigidity of these supports is "spread" and we consider the stability of the beam on a solid elastic base whose coefficient of rigidity is equal to  $k = \frac{K}{a}$ , where  $a$  is the distance between the supports?

212. Using the data in problem 211, show that the acceptance of the "spreading" of the supports' rigidity leads to a substantial error in the value of the beam's Euler force if the rigidity of the supports is equal to the critical rigidity.

#### Stability of Flat Coverings

213. Determine the compressive (critical) stress which a deck covering (Fig. 159) can support with the following initial data:  $l = 8 \text{ m}$ ;  $L = 16 \text{ m}$ ;  $a = 2 \text{ m}$ ;  $b = 0.4 \text{ m}$ ;  $\alpha_1 = \alpha_2 = 0.25$ ;  $I = 34.6 \cdot 10^{-4} \text{ m}^4$ ;  $I = 6.8 \cdot 10^{-6} \text{ m}^4$ ;  $F = 4.3 \cdot 10^{-3} \text{ m}^2$ ;  $\sigma_r = 3000 \text{ kg/cm}^2$ ;  $E = 2 \cdot 10^6 \text{ kg/cm}^2$ .

214. The necessary moment of inertia for the beam of the covering considered in the preceding problem, computed without consideration of the effect of deviation from Hooke's law on

stability, is  $I = 16.6 \cdot 10^{-4} \text{ m}^4$ . How do the support pair coefficients  $\alpha_r = \alpha_s$  of the beams' fastenings change if the deviation from Hooke's law is taken into consideration at the same values of the beams' rigidity and the compressive load?

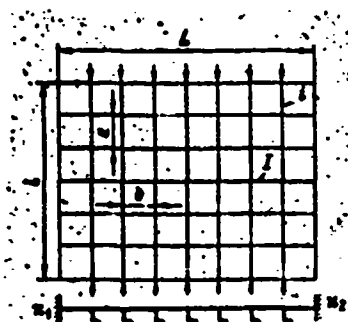


Fig. 159.

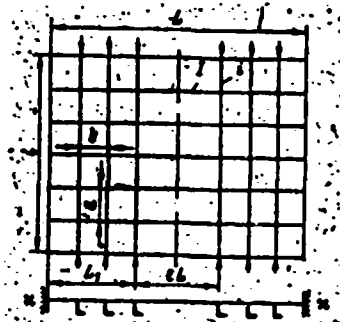


Fig. 160.

215. Determine the necessary moment of inertia  $I$  of a beam in order to provide the covering (Fig. 159) with a critical stress of  $\sigma_{kp} = 2700 \text{ kg/cm}^2$  with the following initial data:  $l = 15 \text{ m}$ ;  $L = 20 \text{ m}$ ;  $a = 2.5 \text{ m}$ ;  $b = 0.4 \text{ m}$ ;  $\alpha_1 = \alpha_6 = 0.75$ ;  $I = 9 \cdot 10^{-6} \text{ m}^4$ ;  $F = 5.2 \cdot 10^{-3} \text{ m}^2$ ;  $\sigma_r = 3000 \text{ kg/cm}^2$ ;  $E = 2 \cdot 10^6 \text{ kg/cm}^2$ .

Compute the moment of inertia  $I$  of the lengthwise beams at which the necessary moment of inertia of the beam can be decreased by 30%. Take the area of the lengthwise rib equal to  $I_r = 0.4 \sqrt{l}$  and its area with the adjacent band  $F = I_r + b l$  in the calculations, where  $t = 0.01 \text{ m}$  is the thickness of the deck of the covering.

216. Find the necessary and critical moments of inertia of the beams for a covering (Fig. 160) at the following initial data:  $l = 12 \text{ m}$ ;  $L = 10 \text{ m}$ ;  $a = 2 \text{ m}$ ;  $b = 0.5 \text{ m}$ ;  $\alpha_1 = \alpha_6 = \frac{L}{2EI}$ ;  $\alpha = 0.4$ ;  $I = 10 \cdot 10^{-6} \text{ m}^4$ ;  $F = 5.75 \cdot 10^{-3} \text{ m}^2$ ;  $\sigma_{kp} = 3600 \text{ kg/cm}^2$ ;  $\sigma_r = 4000 \text{ kg/cm}^2$ ;  $E = 2 \cdot 10^6 \text{ kg/cm}^2$ .

217. Determine the necessary moment of inertia of beams providing the covering with  $\sigma_{kp} = 3000 \text{ kg/cm}^2$  for a covering which consists of a large number of lengthwise beams and three equidistant carlings (Fig. 161). The beams are freely supported on the edges. Also determine the necessary moment of inertia of the beams for a covering without carlings, which are replaced by ordinary lengthwise beams. Given:  $l = 15 \text{ m}$ ;  $L = 12 \text{ m}$ ;  $a = 2 \text{ m}$ ;  $b = 0.4 \text{ m}$ ;  $b_1 = \frac{L}{4} = 3 \text{ m}$ ;  $F = 6.2 \cdot 10^{-3} \text{ m}^2$ ;  $i = 11.2 \cdot 10^{-6} \text{ m}^4$ ;  $F_1 = 11.8 \cdot 10^{-3} \text{ m}^2$ ;  $i_1 = 3.8 \cdot 10^{-4} \text{ m}^4$ ;  $\sigma_r = 4000 \text{ kg/cm}^2$ ;  $\sigma_{kp} = 3600 \text{ kg/cm}^2$ ;  $E = 2 \cdot 10^6 \text{ kg/cm}^2$ .

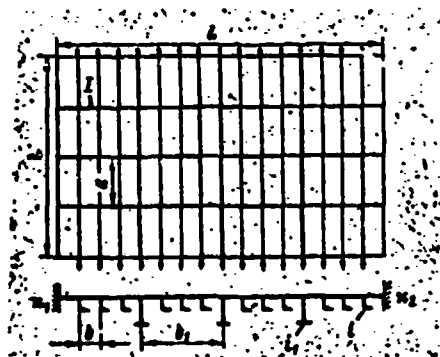


Fig. 161.

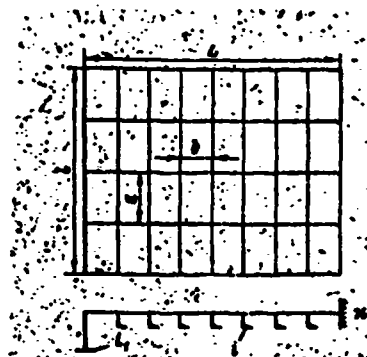


Fig. 162.

218. Determine the value of the compressive stress which the covering in Fig. 162 can support at the following initial data:

$l = 12 \text{ m}$ ;  $L = 3.6 \text{ m}$ ;  $a = 2 \text{ m}$ ;  $b = 0.4 \text{ m}$ ;  $\alpha = 0.5$ ;  $F = 4 \cdot 10^{-3} \text{ m}^2$ ;  $i = 8.1 \cdot 10^{-6} \text{ m}^4$ ;  $F_1 = 10 \cdot 10^{-3} \text{ m}^2$ ;  $i_1 = 2.8 \cdot 10^{-4} \text{ m}^4$ ;  $I = 1.89 \cdot 10^{-4} \text{ m}^4$ ;  $\sigma_r = 4000 \text{ kg/cm}^2$ .

# ANSWERS, INSTRUCTIONS AND SOLUTIONS

$$1. \quad w(x) = -\frac{q_0 l^4}{120EI} \left( \frac{x^5}{l^5} - 10 \frac{x^3}{l^3} + 20 \frac{x^2}{l^2} + 40 \frac{21EIx}{l^3} \right).$$

$$2. \quad w(x) = -\frac{q_0 l^4}{24EI} \left\{ \frac{x}{l} \cdot \frac{a}{1+a} - 2 \frac{x^2}{l^2} \left[ 1 + \frac{1}{4(1+a)} \right] + \frac{3}{2} \cdot \frac{x^3}{l^3} \cdot \frac{1}{1+a} + \frac{x^4}{l^4} \right\}$$

where

$$a = \frac{321EI}{l^3}.$$

$$3. \quad w(x) = -\frac{q_0 l^4}{48EI} \left[ \frac{8\beta}{1+\beta} + \frac{(1-8\beta)}{(1+\beta)} \cdot \frac{x}{l} - \frac{3}{(1+\beta)} \cdot \frac{x^2}{l^2} + 2 \frac{x^3}{l^3} \right], \text{ where } \beta = \frac{34EI}{l^3}.$$

$$4. \quad w(x) = -\frac{Pl^3}{6EI} \left[ \frac{3}{8} (1-x) \frac{x}{l} + \frac{3}{8} x \frac{x^2}{l^3} - \frac{1}{2} \cdot \frac{x^3}{l^3} + \frac{1}{2} \left( \frac{x}{l} - \frac{1}{2} \right)^3 \right]; M_{\text{on}} = -\frac{Pl}{8} x,$$

where

$$x = \frac{1}{1 + \frac{221EI}{l^3}}$$

$$5. \quad \Delta l = 0.152 \frac{l}{EI}; M_{\text{max}} = -\frac{q_0 l^3}{11.65}.$$

$$6. \quad M_{\text{on}} = x \left( \frac{qa^3}{3} + \frac{qa^2 l_1}{4} \right); w_{\text{max}} = -\frac{qa^4}{24EI} (5-4x) + \frac{qa^2 l_1^2}{24EI} (4-3x); M(a) = -x \left[ \frac{qa^3}{6} \left( 1 + \frac{321EI}{a} \right) + \frac{qa^2 l_1}{4} \left( 1 + \frac{221EI}{a} \right) \right], \text{ where } x = \frac{1}{1 + \frac{21EI}{a}}.$$

$$7. \quad M(x) = -2R_0 \left( \frac{x}{l} - \frac{1}{2} \right)$$

$$8. \quad w'(x) = \frac{q_0 l^3}{24EI} \left( 1 - 6 \frac{x^2}{l^2} + 4 \frac{x^3}{l^3} \right).$$

$$9. \quad w(x) = -\frac{q_0 l^4}{360EI} \left( 7 \frac{x}{l} - 10 \frac{x^3}{l^3} + 3 \frac{x^5}{l^5} \right).$$

$$10. \quad w(x) = -\frac{Pl^3}{6EI_1} \left\{ \frac{x^3}{l^3} \left( 3 - \frac{x}{l} \right) - \frac{1}{l_2} \cdot \frac{a^3}{l^3} \left[ 3 - 2 \frac{a}{l} - 6 \frac{x}{a} + \frac{x^2}{a^2} \left( 3 - \frac{x}{l} \right) + 3 \frac{x}{l} - \frac{l_1}{l_2} \left( 1 - \frac{x}{a} \right)^2 \left( 3 - \frac{x+2a}{l} \right) \right] \right\}.$$

11. Solution. The general expression for an elastic beam line can be written in the form

$$w(x) = C_0 + C_1 x + C_2 \frac{x^2}{2} + C_3 \frac{x^3}{6} + \int_0^x \frac{q(x-a)^2}{24EI} + \frac{R(x-a)^2}{6EI}, \quad (1)$$

where  $R$  is the reaction on the middle support (the positive direction coincides with axis  $z$ ). The integration constants and the unknown reaction can be determined from conditions:

$$\text{at } \left. \begin{array}{l} x=0 \quad w=w'=0; \\ x=l \quad w=w'=0; \\ x=a \quad w=0. \end{array} \right\} \quad (2)$$

Making (1) satisfy conditions (2), we have

$$\left. \begin{array}{l} C_0 = C_1 = 0; \\ C_2 \frac{l^2}{2} + C_3 \frac{l^3}{6} + \frac{q(l-a)^2}{24EI} + \frac{R(l-a)^2}{6EI} = 0; \\ C_2 + C_3 l + \frac{q(l-a)^2}{24EI} + \frac{R(l-a)}{EI} = 0; \\ C_2 \frac{a^2}{2} + C_3 \frac{a^3}{6} = 0. \end{array} \right\} \quad (3)$$

Hence we find

$$\begin{aligned} C_2 &= -\frac{q(l-a)^2}{4EI} \frac{1}{4l-a}; \\ C_3 &= -\frac{3}{a} C_2 = \frac{3}{4} \frac{q(l-a)^2}{EI} \frac{1}{a(4l-a)}; \\ R &= -\frac{q(l-a)}{4} \frac{3l^2 + 4al - a^2}{a(4l-a)}. \end{aligned}$$

Therefore, an elastic beam line will have the form

$$w(x) = -\frac{q(l-a)^2}{8EI} \frac{a^2}{4l-a} \left[ \frac{x^2}{a^2} - \frac{x^3}{a^3} + \int_0^x \frac{l^2(3l+a) + a^2(a-5l)}{3a^3} \left( \frac{x-a}{l-a} \right)^2 - \frac{(4l-a)(l-a)}{3a^3} \left( \frac{x-a}{l-a} \right)^3 \right].$$

12.

$$M(x) = \frac{ql^2}{2} \left( \frac{x^2}{l^2} - 1 \right); \quad N(x) = qx.$$

Figure 210 shows the bending moment and shear diagrams.

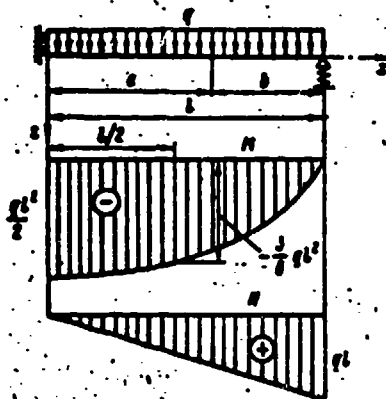


Fig. 210.

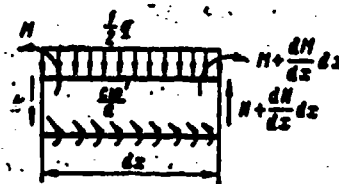


Fig. 211.

13. a)  $\gamma \approx 0,352$ ; b)  $\gamma \approx 0,375$  (the solution of equation  $3\gamma^3 + 6\gamma^2 - 1 = 0$ ).

14.  $x_1 = \frac{7a_1 - 8}{2(a_1 a_2 - 1)}$ ;  $x_2 = \frac{8a_1 - 7}{3(a_1 a_2 - 1)}$ ,  
where  $a_1 = 2 \left( 1 + \frac{32L_1 EI}{l} \right)$ ;  $a_2 = 2 \left( 1 + \frac{32L_2 EI}{l} \right)$ .

15.  $M_1 = \frac{q l^3}{4} \frac{a_1 + l}{(a_1 + 2l)(a_2 + 2l) - l^2}$ ;

$M_2 = \frac{q l^3}{4} \frac{a_2 + l}{(a_1 + 2l)(a_2 + 2l) - l^2}$ , where  $a_i = 6EI/l_i$ , ( $i = 1, 2$ ).

16.  $M_A = \frac{q l^3}{4} \frac{l^2}{(a + 2l)(2a + 3l)}$ ;  $M_B = \frac{q l^3}{2} \frac{a + l}{2a + 3l}$ ;  $M_C = \frac{q l^3}{4} \frac{4a + 7l}{(a + 2l)(2a + 3l)}$ ,

where

$$a = 6EI/2l.$$

17.

$$x = c = 0,59l.$$

18.

$$\frac{l}{l} = 2.$$

19.

$$\frac{l}{l} \approx 1,29.$$

20. Solution. The elastic beam lines are identical; the braces' bending point is halfway up their height. Because of this, each of the beams bends under a load with intensity of  $0.5 q$  and a bending moment whose intensity is equal to  $\frac{c}{a}$ , where  $c = \frac{6EI}{l}$  is the proportionality constant between the angle of rotation of

the final cross section of the brace and the bending moment applied to this section.

Composing the equilibrium equation for an element isolated from the shaft (Fig. 211), we obtain

$$\frac{dM(x)}{dx} = N(x) + \frac{c}{a} w'; \quad \frac{dN(x)}{dx} = \frac{1}{2} q; \quad \frac{d^2 M(x)}{dx^2} = \frac{1}{2} q + \frac{c}{a} w'$$

or

$$EI w^{IV} - \frac{6}{a} \frac{EI}{l} w' = \frac{1}{2} q.$$

We can write the integral of this equation in the form

$$w(x) = -\frac{qal}{24EI} x^3 + A + Bx + C \operatorname{ch} ax + D \operatorname{sh} ax,$$

where

$$a = \frac{1}{l} \sqrt{6 \frac{l}{l} \frac{l}{a}}.$$

Determining the integration constants A, B, C, D from the boundary conditions

$$x=0; \quad w' = w'' = 0; \quad x = \frac{l}{2}; \quad w = w' = 0,$$

we will find

$$w(x) = -\frac{qal^3}{24EI} \left( \frac{u^3 - 2}{u^3} - \frac{x^3}{l^3} + \frac{2}{u^3} \frac{\operatorname{ch} ux}{\operatorname{ch} u} \right),$$

where

$$u = \alpha.$$

$$21. \quad w_{\max} = \frac{1}{128} \frac{Pl^3}{EI}; \quad 2) \quad w_{\max} = -\frac{5}{384} \frac{Pl^3}{EI}.$$

$$22. \quad w(x) = w_1 + w_2 = \frac{Pl^3}{6EI} (1-c) \left\{ \frac{6EI}{G\omega l^3} \frac{x}{l} - \frac{x^3}{l^3} + \left[ 1 - \left( \frac{1-c}{l} \right)^3 \right] \frac{x}{l} + \right.$$

$$\left. \frac{1}{6} \frac{x-c}{l-c} \left[ \left( \frac{x-c}{l} \right)^3 - \frac{EI}{G\omega l^3} \right] \right\}.$$

23. The elastic line will be antisymmetric relative to the middle cross section of the beam. The origin of the coordinates is taken in the middle cross section of the beam, axis x is directed toward the right,

$$w(x) = w_1 + w_2 = \frac{q_0^2}{24EI} \left[ \frac{x^5}{5} - 2 \frac{x^3}{3} - 12 \frac{EI}{G \omega^2} \frac{x}{5} + 1 + 12 \frac{EI}{G \omega^2} \right]$$

$$24. \quad w(x) = \frac{q_0^2}{24EI} \left[ \frac{x^5}{5} - \frac{x}{1} \left[ \frac{12EI}{G \omega^2} - \frac{3}{2a} \left( 1 + 12 \frac{AEI}{\rho} \right) \right] - \frac{\beta}{a} \left( \frac{1}{2} \frac{x^3}{\rho} - \frac{3EI}{G \omega^2} \right) \right];$$

$$M(x) = \frac{q_0^2}{2} \left[ \frac{x^3}{3} - \frac{1}{4} \frac{\beta}{a} \frac{x}{1} + \frac{1}{4a} \left( 1 + \frac{12AEI}{\rho} \right) \right],$$

where

$$\alpha = 1 + \frac{3EI}{G \omega^2} + \frac{3AEI}{\rho}; \quad \beta = 5 + 12 \frac{EI}{G \omega^2} + 24 \frac{AEI}{\rho}$$

$$25. \quad w(x) = w_1(x) + w_2(x) = \frac{q_0^2}{384EI} \left\{ 1 - 8 \left( \frac{x}{l} \right)^2 + 16 \left( \frac{x}{l} \right)^4 + \frac{18EI}{G \omega^2} \left[ 1 - 4 \left( \frac{x}{l} \right)^2 \right] \right\}.$$

The origin of the coordinates was taken half-way down the length of the beam.

$$26. \quad 1) \quad w = \frac{q_0^2}{384EI} \left( 1 + \frac{125}{5-4\kappa} \frac{l}{\omega^2} \right) (5-4\kappa);$$

$$2) \quad w = \frac{P l^3}{192EI} \left( 1 + \frac{125}{4-3\kappa} \frac{l}{\omega^2} \right) (4-3\kappa).$$

Poisson's ratio  $\nu = 0.3$ .

27. Instructions. Use the results of the solution to problem 26.

$$R = 0.5Q \frac{1}{1 + \frac{k_1}{k_2} \left( \frac{l_1}{l_2} \right)^2 \frac{l_2}{l_1}},$$

where

$$k_1 = 1 + 125 \frac{l_1}{\omega_1 l_1^2}; \quad k_2 = 1 + 125 \frac{l_2}{\omega_2 l_2^2}.$$

$$28. \quad 1) \quad M_{\text{on}} = - \frac{q_0^2}{2} \frac{1 - 6 \frac{EI}{G \omega^2}}{1 + 3 \frac{EI}{G \omega^2}};$$

$$2) \quad M_{x=0} = - \frac{q_0^2}{30} \frac{1}{a} \left( 1 + 15 \frac{EI}{G \omega^2} \right);$$

$$M_{x=l} = - \frac{q_0^2}{20} \frac{1}{a} \left( 1 + 10 \frac{EI}{G \omega^2} \right);$$

$$3) \quad M_{\text{on}} = \frac{6EI}{l^2} \frac{l}{a}, \quad \text{where } a = 1 + 12 \frac{EI}{G \omega^2}.$$

$$29. \quad w_{\text{max}} = \frac{q_0^2}{482EI} \left( 1 + 45.5 \frac{EI}{G \omega^2} \right).$$

30. The pliability factor at point D

$$A_D = A_2 + \frac{q_1}{48EI_1} + \frac{q_2}{12EI_2}$$

the pliability factor at point C

$$A_C = \frac{192EI_1}{q_1} + \frac{3EI_2}{q_2} \left( \frac{1}{1 + \frac{3EI_2A_1}{q_1}} \right)$$



Fig. 212.

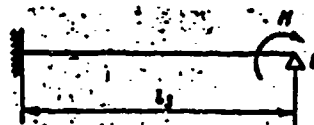


Fig. 213.

31. Solution. Since point A of beam I-I is immobile, this beam's angle of rotation at point A is determined from the beam's design diagram (Fig. 212) from the system of equations

$$\psi_A = \frac{M_1 l_1}{3EI_1} - \frac{M_2 l_2}{3EI_2}; \quad M_1 + M_2 - M = 0,$$

hence

$$M_1 = M_2 \frac{l_2}{l_1} \frac{l_1}{l_2}; \quad M_2 \left( 1 + \frac{l_2}{l_1} \frac{l_1}{l_2} \right) = M.$$

Therefore,

$$\psi_A = \frac{M l_2}{3EI_2 \left( 1 + \frac{l_2}{l_1} \frac{l_1}{l_2} \right)}$$

The pliability factor

$$\eta_A = \frac{l_2}{3EI_2 \left( 1 + \frac{l_2}{l_1} \frac{l_1}{l_2} \right)}$$

In order to determine the pliability factor of beam AB in cross section B, we find the angle of rotation at the free support of the beam (Fig. 213) from the concentrated moment M applied to the free support

$$\theta_B = -\frac{Ml}{4EI}.$$

Therefore, the pliability factor of the elastic fixing

$$\alpha_B = -\frac{l}{4EI}.$$

32. In the middle of the span

$$M_1 = \frac{ql^2}{12} \frac{1 + \frac{621EI}{l} + \frac{48AEI}{\rho} \left(1 + \frac{321EI}{l}\right)}{1 + \frac{421EI}{l} + \frac{24AEI}{\rho} \left(1 + \frac{21EI}{l}\right)}.$$

In the support cross sections

$$M_{\text{on}} = -\frac{1}{1 + \frac{221EI}{l}} \left( \frac{ql^2}{6} - M_1 \right).$$

33.  $\frac{q_1}{q_0} = 1.57.$

34. The sagging of the elastic support  $l_F = \frac{q^N}{96EI}$ . The vertical displacement of joint G  $l_G = \frac{5}{84} \frac{q^N}{EI}$ .

The bending moment and shear diagrams are given in Fig. 214.

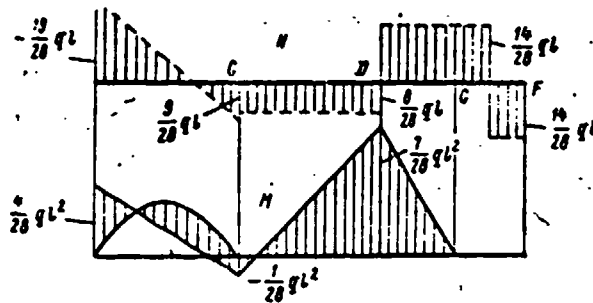


Fig. 214.

35.  $I = \frac{\gamma^2 F_2^2}{24 (F_1 - F_2) E}.$

36.  $M = -\frac{1}{30} ql^2.$

37.

$$A = \frac{P}{12EI}.$$

38. Solution. The pliability factor of the elastic fixing at the left end of the beam

$$\alpha_0 = \frac{M_0}{EI_0} = \frac{f_1}{3EI_0} = \frac{0.8l}{3E \cdot 6I_0} = \frac{l}{36EI_0}.$$

The pliability factors of the first and second intermediate supports

$$\begin{aligned} A_1 &= \frac{f_2}{3EI_0} = \frac{(0.5l)^2}{3E \cdot 2.08I_0} = \frac{P}{56EI_0}, \\ A_2 &= \frac{c^2(l-c)^2}{3EI_0} = \frac{(0.4l)^2(0.6l)^2}{3E \cdot 1.92I_0} = \frac{P}{100EI_0}. \end{aligned}$$

The equations for the equality of the angles of rotation at the supports can be written in the form

$$\left. \begin{aligned} 2L_0 M_0 &= -\frac{M_1 l}{3EI_0} - \frac{M_1 l}{6EI_0} + \frac{q l^3}{24EI_0} + \frac{f_1}{l}; \\ \frac{M_1 l}{6EI_0} + \frac{M_1 l}{3EI_0} - \frac{q l^3}{24EI_0} + \frac{f_1}{l} &= -\frac{M_2 \cdot 1.5l}{3EI_0} - \\ &\quad - \frac{M_2 \cdot 1.5l}{6EI_0} + \frac{q(1.5l)^3}{24EI_0} + \frac{f_2 - f_1}{1.5l}; \\ \frac{M_2 \cdot 1.5l}{6EI_0} + \frac{M_2 \cdot 1.5l}{3EI_0} - \frac{q(1.5l)^3}{24EI_0} + \frac{f_2 - f_1}{1.5l} &= \\ &= -\frac{M_3 l}{3EI_0} - \frac{M_3 l}{6EI_0} + \frac{q l^3}{24EI_0} - \frac{f_2}{l}; \\ \frac{M_3 l}{6EI_0} + \frac{M_3 l}{3EI_0} - \frac{q l^3}{24EI_0} - \frac{f_2}{l} &= 0. \end{aligned} \right\} \quad (4)$$

Here  $f_1$  and  $f_2$  (sagging of sections 1 and 2) are related to the support moments and the load by equations

$$\left. \begin{aligned} f_1 &= -A_1 \left( \frac{M_0 - M_1}{l} - \frac{q l}{2} - P_1 - \frac{M_1 - M_0}{1.5l} - \frac{q l \cdot 1.5l}{2} \right); \\ f_2 &= -A_2 \left( \frac{M_1 - M_2}{1.5l} - \frac{q l \cdot 1.5l}{2} - P_2 - \frac{M_2 - M_1}{l} - \frac{q l}{2} \right). \end{aligned} \right\} \quad (5)$$

Substituting (5) in (4) and using the previously found values for the pliability factors of the elastic fixing and the elastic

supports ( $M_0$ ,  $A_1$  and  $A_2$ ), we obtain the following system of equations:

$$\begin{aligned} 1) & 2,32M_0 + 0,8M_1 + 0,08M_2 = 0,46qP; \\ 2) & 0,8M_0 + 5,36M_1 + 1,3M_2 + 0,04M_3 = 0,83qP; \\ 3) & 0,08M_0 + 1,3M_1 + 5,22M_2 + 0,9M_3 = 1,01qP; \\ 4) & 0,04M_1 + 0,9M_2 + 2,06M_3 = 0,39qP. \end{aligned}$$

The system of equations is solved according to the Gauss system in tabular form:

Table 1.

Equation number	$M_0$	$M_1$	$M_2$	$M_3$	$qP$	Sum of coefficients
1	2,32	0,80	0,08	—	0,46	3,66
2	0,80	5,36	1,30	0,04	0,83	8,33
3	0,08	1,30	5,22	0,90	1,01	8,51
4	—	0,04	0,90	2,06	0,39	3,39
I	1	0,354	0,034	—	0,198	1,557
2	0,8	5,36	1,30	0,04	0,83	8,33
I · (-0,9)	-0,8	-0,276	-0,027	—	-0,159	-1,262
$\Sigma = 2^{(1)}$	—	5,084	1,273	0,04	0,671	7,068
II	—	1,00	0,251	0,008	0,132	1,391
3	0,08	1,30	5,22	0,90	1,01	8,51
I · (-0,08)	-0,08	-0,028	-0,003	—	-0,016	-0,127
II · (-1,272)	—	-1,272	-0,32	-0,010	-0,168	-1,77
$\Sigma = 3^{(2)}$	—	—	4,8	0,89	0,826	6,613
III	—	—	1,00	0,182	0,169	1,351
4	—	0,04	0,90	2,06	0,39	3,39
II · (-0,04)	—	-0,04	-0,01	-0,007	-0,005	-0,055
III · (-0,89)	—	—	-0,89	-0,182	-0,150	-1,202
$\Sigma = 4^{(3)}$	—	—	—	1,898	0,235	2,133
IV	—	—	—	1	0,124	—
III	—	—	1	—	0,146	—
II	—	1	—	—	0,095	—
I	1	—	—	—	0,160	—

Thus,  $M_0 = 0,160qP$ ;  $M_1 = 0,095qP$ ;  $M_2 = 0,146qP$ ;  $M_3 = 0,124qP$ .

The shear forces on the beam supports: on the left support

$$N_0 = -\frac{M_0 - M_1}{l} - 0.5ql = -0.565ql; \text{ to the left of the first support}$$

$$N_{1-0} = -\frac{M_0 - M_1}{l} + 0.5ql = 0.435ql;$$

to the right of the first support

$$N_{1+0} = -\frac{M_1 - M_2}{1.5l} - 0.75ql = -0.716ql;$$

to the left of the second support

$$N_{2-0} = -\frac{M_1 - M_2}{1.5l} + 0.75ql = 0.784ql;$$

to the right of the second support

$$N_{2+0} = -\frac{M_2 - M_3}{l} - 0.5ql = -0.522ql;$$

at the right end of the beam

$$N_3 = -\frac{M_2 - M_3}{l} + 0.5ql = 0.478ql.$$

Figure 215 shows the bending moment and shear diagram of the beam in question.

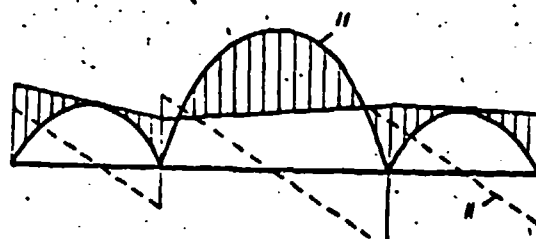


Fig. 215.

39. For the beam shown in Fig. 24, support moments  $M_1 = 0.046ql^2$ ;  $M_2 = 0.015ql^2$ ;  $M_3 = 0.022ql^2$ ; the shear forces in the support sections

$$N_1 = -0.31ql; N_{2-0} = 0.064ql; N_{2+0} = -0.068ql; N_3 = 0.232ql.$$

Figure 216 shows the bending moment and shear diagrams.

For the beam shown in Fig. 25, support moments  $M_1 = -0.135ql^2$ .

$M_2 = 0.013ql^2$ ;  $M_3 = 0.120ql^2$ ; the shear forces in the support sections

$$N_{1-0} = -0.62Q; N_{2-0} = 0.38Q; N_{2+0} = -0.14Q; N_{3-0} = 0.36Q.$$

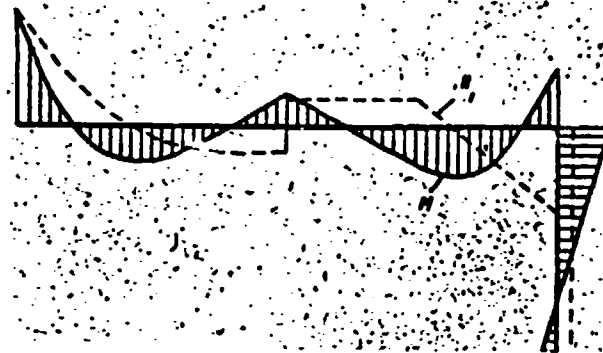


Fig. 216.

Figure 217 shows the bending moment and shear diagrams.

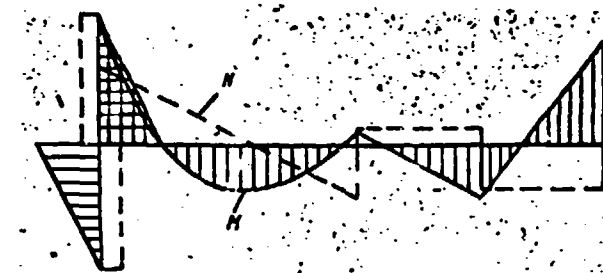


Fig. 217.

40. The support moments:  $M_0 = -0.0096ql^2$ ;  $M_1 = 0.677ql^2$ ;  $M_2 = -0.1185ql^2$ .

The moments in the middle of the spans

$$M_{0.5} = -0.117ql^2; M_{1.5} = -0.47ql^2; M_{2.5} = -0.184ql^2.$$

The shear forces on the supports  $N_0 = -0.213ql$ ;  $N_{1.0} = 1.79ql$ ;  $N_{2.0} =$

$$-2.4ql; N_{3.0} = 0.6ql; N_{4.0} = -0.33ql; P'_{3.0} = 0.67ql.$$

Figure 218 shows the bending moment and shear diagrams.

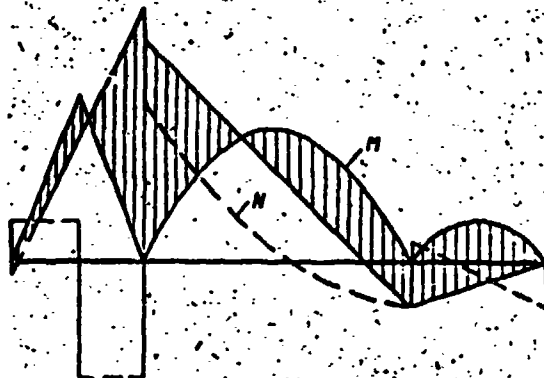


Fig. 218.

41. Support moments  $M_0 = 0.067ql^2$ ;  $M_1 = 0.133ql^2$ ;  $M_2 = 0.04ql^2$ ;  $M_3 = 0.39ql^2$ .

Shear forces  $N_0 = -0.935ql$ ;  $N_{1-0} = 1.045ql$ ;  $N_{1-1} = -0.813ql$ ;

$N_{2-0} = 0.687ql$ ;  $N_{2-1} = -0.43ql$ ;  $N_{3-0} = 1.57ql$ .

Fig. 219 shows the bending moment and shear diagrams.

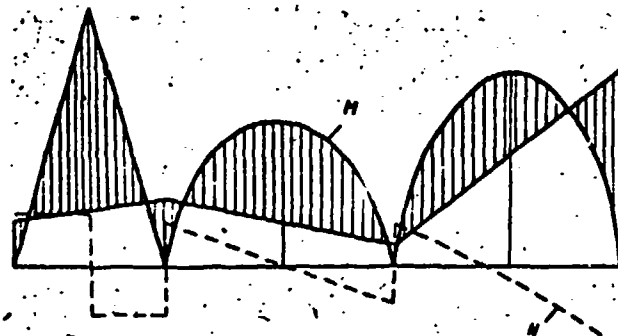


Fig. 219.

42.

$$\begin{aligned} M_0 &= 0.156Ql_0; \\ M_1 &= 0.00072Ql_0; \\ M_2 &= 0.031Ql_0; \\ I_1 &= \frac{Ql_0^3}{1045EI_0}. \end{aligned}$$

43.

44.

where  $Q_1$  is the load on the upper beam;  $Q_0$  is the load on the lower beam.

45.

46. Instructions. Use the theorem of three moments.  
Ratio  $\alpha = \frac{h}{l}$  is determined by equation  $\alpha^4 + 2.5\alpha^2 - 2\alpha - \frac{1}{3} = 0$ , the  
solution to which  $\alpha \approx 0.77$ .

47.

48. Instruct ons. Use the hypothetical support method. The support moment - determined by the equation

$$\mu_{\text{en}} = \frac{q^2}{12} \left[ \frac{I_2}{I} + 2 \left( \frac{c}{I} \right)^3 + 6 \left( 1 + \frac{c}{I} \right) \frac{c}{I} \left( \frac{c}{I} + \frac{I_2}{I} \right) \right] \frac{1}{2 \frac{c}{I} + \frac{I_2}{I}}.$$

49.

50.

51.

b)

52. Figure 220 shows the bending moment diagram.

53. Figure 221 shows the bending moment diagram.

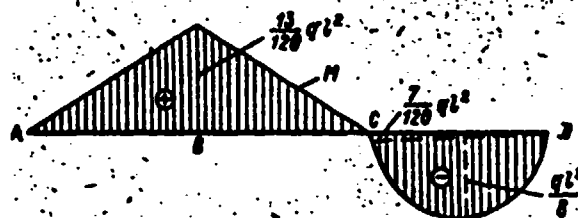


Fig. 220.

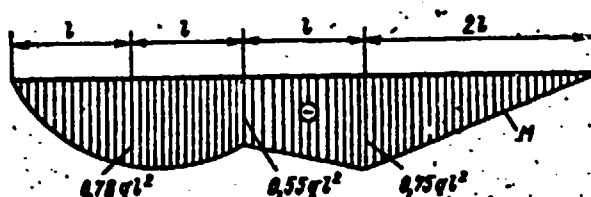


Fig. 221.

54. 
$$\kappa_{cp} = 0.6 + 0.3 \frac{q_1}{q} \left( \frac{l_1}{l} \right)^2 + 0.3 \frac{q_2}{q} \left( \frac{l_2}{l} \right)^2.$$

55. The moments on the second and third supports

$$M_2 = -\frac{q_0 l_0^2}{12} \left[ 0.666 + 0.466 \frac{q_1}{q_0} \left( \frac{l_1}{l_0} \right)^2 - 0.133 \frac{q_2}{q_0} \left( \frac{l_2}{l_0} \right)^2 \right];$$

$$M_3 = -\frac{q_0 l_0^2}{12} \left[ 0.666 + 0.466 \frac{q_1}{q_0} \left( \frac{l_1}{l_0} \right)^2 - 0.133 \frac{q_1}{q_0} \left( \frac{l_1}{l_0} \right)^2 \right].$$

The moments at the fixing

$$M_1 = -\frac{q_1 l_1^2}{12} \left[ 1.267 - 0.333 \frac{q_2}{q_1} \left( \frac{l_2}{l_1} \right)^2 + 0.067 \frac{q_1}{q_1} \left( \frac{l_1}{l_1} \right)^2 \right];$$

$$M_4 = -\frac{q_2 l_2^2}{12} \left[ 1.267 - 0.333 \frac{q_2}{q_2} \left( \frac{l_2}{l_2} \right)^2 + 0.067 \frac{q_1}{q_2} \left( \frac{l_1}{l_2} \right)^2 \right];$$

$$\kappa_{cp} = \frac{M_2 + M_3}{q_0 l_0^2} = 0.67 + 0.165 \frac{q_1}{q_0} \left( \frac{l_1}{l_0} \right)^2 + 0.165 \frac{q_2}{q_0} \left( \frac{l_2}{l_0} \right)^2$$

56. Figure 222 shows the bending moment diagram.

57.

$$\Delta = \frac{P}{24ET}.$$

58.

$$R_2 = -R_3 = \frac{6EI}{l} \delta.$$

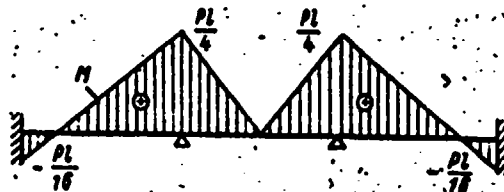


Fig. 222.

59. 
$$J = \frac{PP^2}{4ET}.$$

60. 
$$\frac{q^2}{q} = 0.75.$$

61. 
$$x = \frac{1 + \frac{2}{15} \frac{HC}{B^2} \frac{Q_1}{Q} \frac{I}{T} \left[ 8 + 9 \frac{H-C}{H} + 3 \frac{(H-C)^2}{H^2} \right]}{1 + \frac{2}{3} \frac{H}{B} \frac{I}{T}}.$$

62. 
$$I = \frac{TI^2 \cos^2 \alpha \sin^2 \alpha}{3EI_1} \left( \cos \alpha + \frac{I_1}{I_2} \sin \alpha \right).$$

63. 
$$M_1 = -0.16ql^2; M_2 = 0.21ql^2; M_3 = -0.13ql^2.$$

Figure 223 shows the bending moment diagram.

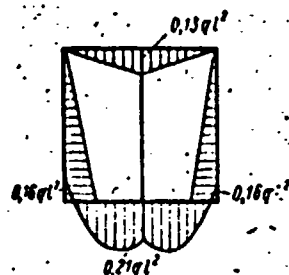


Fig. 223.

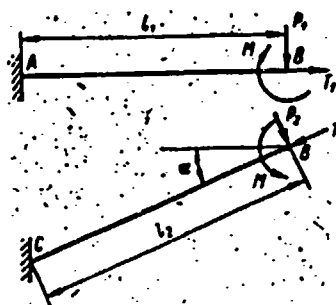


Fig. 224.

64. 
$$M_1 = -0.0254ql^2; M_2 = 0.00725ql^2; M_3 = -0.1123ql^2; M_4 = -0.00367ql^2.$$

65. Solution. Separating the rods in bundle B and loading them with the forces applied to the ends of the rods (Fig. 224),

we will write the conditions for equilibrium and the equality of the displacements of the ends of the rods. The equilibrium conditions:

$$P_1 + P_2 \cos \alpha + T_2 \sin \alpha = P; P_2 \sin \alpha + T_1 - T_2 \cos \alpha = 0.$$

The conditions for the equality of the displacement of the ends of the rods:

$$\begin{aligned} \frac{P_1 l_1^3}{3EI} + \frac{M l_1^2}{2EI} &= \left( \frac{P_2 l_2^3}{3EI} - \frac{M l_2^2}{2EI} \right) \cos \alpha + \frac{T_2 l_2}{EF} \sin \alpha; \\ \frac{T_1 l_1}{EF} &= \left( \frac{P_2 l_2^3}{3EI} - \frac{M l_2^2}{2EI} \right) \sin \alpha - \frac{T_2 l_2}{EF} \cos \alpha; \\ \frac{P_1 l_1^3}{3EI} + \frac{M l_1^2}{EI} &= \frac{P_2 l_2^3}{2EI} - \frac{M l_2^2}{EI}. \end{aligned}$$

From the joint solution of the system of equations we find:

$$T_1 = 1.65P; T_2 = 1.93P; P_1 = 0.037P; P_2 = 0.017P; M = -3.1 \cdot 10^{-3} P l_1.$$

The approximate solution to this problem can be obtained in the following manner. Without consideration of dilatational-compressive and bending strain of the rods

$$T_1 = \frac{P}{\lg \alpha} = 1.73P; T_2 = \frac{P}{\sin \alpha} = 2P.$$

In order to determine the bending moments in the rods of an outrigger assembly, one should find the vertical displacement of the bundle B due to extension of rod AB and compression of rod BC. This displacement can be determined from the expression

$$l_{BA} = \frac{T_1 l_1}{EF \lg \alpha} + \frac{T_2 l_2}{EF \sin \alpha} = \frac{1}{EF} [1.73 P l_1 + 2 P l_2] = \frac{7.64 P l_1}{EF}.$$

The displacement of the end of rod B perpendicular to the axis of rod BC

$$l_{BC} = l_{BA} \cos \alpha = \frac{7.64 P l_1}{EF} 0.865 = 0.66 \frac{P l_1}{EF}.$$

The forces which act on the ends of rods AB and BC (see Fig. 224) are determined in this case from the equations for the

equality of the displacements of the rod ends:

$$\frac{P_1 l_1^3}{3EI} + \frac{M_1^2}{2EI} = l_{BA}, \quad \frac{P_2 l_2^3}{3EI} + \frac{M_2^2}{2EI} = l_{BC}$$

$$\frac{P_1 l_1^2}{2EI} + \frac{M_1}{EI} = \frac{P_2 l_2^2}{2EI} + \frac{M_2}{EI}$$

Substituting the values of  $f_{BA}$  and  $f_{BC}$  in these equations and solving them, we will find:  $P_1 = 0.025P$ ;  $P_2 = 0.0145P$ ;  $M = -1.23 \cdot 10^{-3} P l_1$ .

If the results of both solutions are compared, it turns out that the simultaneous consideration of the dilatational-compressive and bending strains of the rod has a substantial effect on the value of the bending moments in the rods when determining the stresses in the rods of the outrigger assembly in question.

66.

$$M_1 = \frac{ql^3}{4} \cdot \frac{3 + \frac{1}{2} \frac{l}{T} \left( \frac{L}{l} \right)^3}{7 + \frac{1}{4} \frac{l}{T} \left( \frac{L}{l} \right)^3}$$

67. For the assemblies depicted in Fig. 51,  $M_1 = -0.047ql^2$ ;  $M_2 = 0.011ql^2$ ;  $M_3 = -0.0106ql^2$ ;  $M_4 = 0.052ql^2$ . For the assemblies shown in Fig. 52,  $M_1 = 0.425Pl + 0.05Ql$ ;  $M_2 = -0.075Pl + 0.05Ql$ ;  $M_3 = 0$ .

68. For the assemblies shown in Fig. 53,  $M_1 = 0.041ql^2$ ,  $M_2 = -0.306ql^2$ ;  $N_{1-1} = -0.5ql$ ,  $N_{1-2} = 0.24ql$ ,  $N_{2-1} = 0.04ql$ ,  $N_{2-2} = N_{3-1} = -0.02ql$ . The bending moment and shear diagrams are shown in Fig. 225.



Fig. 225.

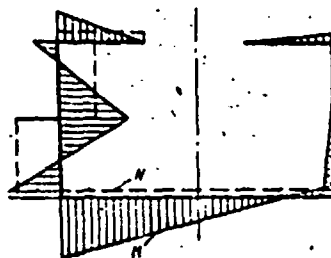


Fig. 226.

For the assemblies shown in Fig. 54,  $N_{1-1} = N_{1-2} = 0.0685P$ ,  $N_{1-3} = -0.55P$ ,  
 $N_{1-4} = 0.45P$ ,  $N_{2-3} = -0.13P$ ,  $N_{2-4} = -0.023P$ ,  $N_{3-2} = 0.009P$ ,  $M_1 =$   
 $-0.0674Pl$ ,  $M_2 = 0.0361Pl$ ,  $M_3 = -0.011Pl$ ,  $M_4 = -0.002Pl$ .

Fig. 226 shows the bending moment and shear diagrams.

69.  $M_1 = M_2 = 0.037ql^2$ ,  $M_3 = M_4 = -0.0185ql^2$ ,  $R = 0.174ql$ ,  $N_{1-1} =$   
 $= -N_{1-2} = -0.413ql$ ,  $N_{1-3} = 0.28ql$ ,  $N_{2-1} = 0.11ql$ ,  $N_{2-2} = -N_{2-3} =$   
 $= -0.067ql$ .

The bending moment and shear diagrams are shown in Fig. 227.

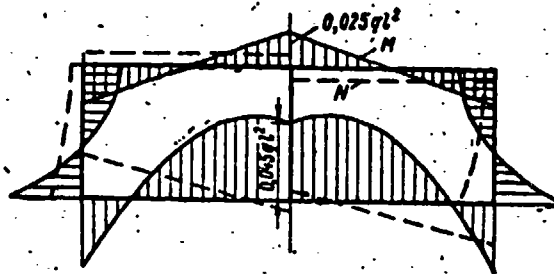


Fig. 227.

70. For the assemblies shown in Fig. 56,  $M_1 = 0.01ql^2$ ,  $M_2 = 0.066ql^2$ ,  $M_3 =$   
 $= 0.091ql^2$ ,  $M_4 = -0.005ql^2$ . The bending moment diagram is shown  
in Fig. 228.

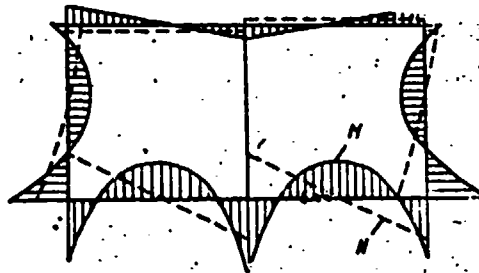


Fig. 228.

For the assemblies depicted in Fig. 57,

$M_{1-1} = -M_{1-2} = 0.141ql^2$ ,  $M_{2-1} = -0.024ql^2$ ,  $M_{2-2} = 0.045ql^2$ ,  $M_{2-3} =$   
 $= -0.021ql^2$ ,  $M_{3-1} = 0.127ql^2$ ,  $M_{3-2} = -0.186ql^2$ ,  $M_{4-1} = 0.061ql^2$ ,  
 $M_{1-3} = 0.0165ql^2$ ,  $M_{1-4} = -0.0165ql^2$ ,  
 $M_{2-4} = 0.045ql^2$ ,  $M_{3-4} = -0.066ql^2$ ,  $M_{4-2} = 0.063ql^2$ .

Figure 229 shows the bending moment diagram.

For the assembly shown in Fig. 58,

$M_{1-1} = -0.096ql^2$ ,  $M_{1-2} = -M_{1-3} = 0.148ql^2$ ,  $M_{2-1} = -0.066ql^2$ ,  $M_{2-2} =$   
 $= -0.008ql^2$ ,  $M_{2-3} = 0.075ql^2$ ,  $M_{3-1} = -M_{3-2} = 0.063ql^2$ ,  $M_{3-3} = 0.087ql^2$ .

The bending moment diagram is shown in Fig. 230.

Instructions. When determining the bending moments in the units of the assemblies shown in Fig. 59, divide the load into the symmetrical and axisymmetrical parts.

The moments in the assembly units are equal to:

$$\begin{aligned} M_{1-2} &= -0.006qR^2; M_{2-1} = -0.003qR^2; M_{2-3} = -M_{1-4} = -0.071qR^2; M_{3-2} = 0.009qR^2; \\ M_{4-3} &= -0.079qR^2; M_{5-4} = 0.067qR^2; M_{6-5} = 0.012qR^2; M_{7-6} = 0.34 \cdot 10^{-2}qR^2; M_{8-7} = \\ &= -0.97 \cdot 10^{-2}qR^2; M_{9-8} = 0.63 \cdot 10^{-2}qR^2; M_{10-9} = -M_{11-10} = 0.3 \cdot 10^{-2}qR^2; M_{12-11} = \\ &= -0.24 \cdot 10^{-2}qR^2; M_{13-12} = 0.29 \cdot 10^{-2}qR^2; M_{14-13} = 0.5 \cdot 10^{-2}qR^2; M_{15-14} = -M_{16-15} = \\ &= -0.4 \cdot 10^{-2}qR^2. \end{aligned}$$

The bending moment diagram is shown in Fig. 231.

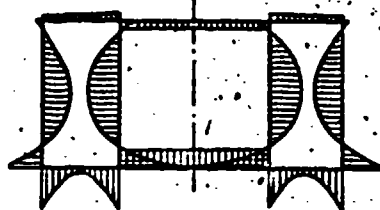


Fig. 229.



Fig. 230.

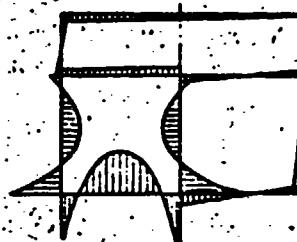


Fig. 231.

$$\begin{aligned} 71. M_{1-2} &= -0.247qR^2 + 0.164q_1R^2; M_{2-1} = -0.041qR^2 + 0.124q_1R^2; M_{2-3} = \\ &= -0.045qR^2 - 0.04q_1R^2; M_{3-2} = 0.086qR^2 - 0.086q_1R^2; M_{3-4} = -0.438qR^2 + \\ &+ 0.52q_1R^2; M_{4-3} = -0.264qR^2 + 0.181q_1R^2; M_{4-5} = 0.172qR^2 - 0.089q_1R^2; M_{5-4} = \\ &= 0.096qR^2 - 0.096q_1R^2; \\ 72. M_{1-2} &= -0.583qR^2; M_{2-1} = -0.417qR^2; M_{2-3} = M_{3-2} = -0.250qR^2; M_{3-4} = \\ &= -0.063qR^2; M_{4-3} = 0. \end{aligned}$$

$$\begin{aligned} 73. M_{1-2} &= -M_{1-4} = -7.05 \cdot 10^{-2}qR^2; M_{2-1} = 1.37 \cdot 10^{-2}qR^2; M_{2-3} = -0.5 \times \\ &\times 10^{-2}qR^2; M_{3-2} = -0.87 \cdot 10^{-2}qR^2; M_{3-4} = -M_{4-3} = -0.12 \cdot 10^{-2}qR^2; M_{4-5} = \\ &= M_{5-4} = -\frac{1}{2}M_{4-3} = -6.46 \cdot 10^{-2}qR^2; M_{5-6} = M_{6-5} = \frac{1}{2}M_{4-3} = 1.62 \cdot 10^{-2}qR^2; \\ M_{6-7} &= -0.54 \cdot 10^{-2}qR^2; M_{7-6} = 0.72 \cdot 10^{-2}qR^2; M_{7-8} = -0.18 \cdot 10^{-2}qR^2; M_{8-7} = \\ &= 0.08 \cdot 10^{-2}qR^2; M_{9-8} = -0.09 \cdot 10^{-2}qR^2; M_{10-9} = 0.01 \cdot 10^{-2}qR^2. \end{aligned}$$

$$74. w(0) = \frac{P}{4\alpha^3 EI}; w'(0) = -\frac{P}{4\alpha^2 EI}.$$

$$75. w(0) = \frac{9R_0}{4\alpha^3 EI}; w'(0) = -\frac{9R_0}{2\alpha^2 EI}.$$

$$76. w(x) = \frac{P}{16\alpha^3 EI} e^{-\alpha x} (\cos \alpha x + \sin \alpha x); w(0)_{\max} = \frac{P}{16\alpha^3 EI}.$$

$$77. w(x) = \frac{q}{k} [1 - e^{-\alpha x} (\cos \alpha x + \sin \alpha x)]; R_0 = \frac{q}{\alpha}; M_{0n} = \frac{q}{2\alpha^2}.$$

$$78. a) w(0) = 0.0086 \frac{q l^3}{EI}; w\left(\pm \frac{l}{2}\right) = \mp 0.0254 \frac{q l^3}{EI}; M(0) = -0.074 q l^2;$$

$$N\left(\pm \frac{l}{2}\right) = \pm 6.339 q l.$$

$$b) w(0) = 0.0127 \frac{P l^3}{EI}; w\left(\pm \frac{l}{2}\right) = \mp 0.0369 \frac{P l^3}{EI}; M(0) = -0.169 P l;$$

$$N\left(\pm \frac{l}{2}\right) = \pm 0.224 P.$$

$$c) w'(0) = -0.251 \frac{R_0 l}{EI}; w'(l) = 0.088 \frac{R_0 l}{EI};$$

$$N(0) = -1.898 \frac{R_0}{l}; N(l) = -0.274 \frac{R_0}{l}.$$

$$d) w'(0) = 0.029 \frac{q l^2}{EI}; M(l) = 0.1 q l^2; N(0) = -0.312 q l; N(l) = 0.529 q l.$$

$$e) w(0) = 0.0023 \frac{q l^3}{EI}; M(0) = -0.0366 q l^2;$$

$$M\left(\frac{l}{2}\right) = 0.075 q l^2; N\left(\pm \frac{l}{2}\right) = \pm 0.46 q l.$$

$$f) w(0) = 0.00463 \frac{P l^3}{EI}; M\left(\frac{l}{2}\right) = 0.109 P l;$$

$$M(0) = -0.115 P l; N\left(\pm \frac{l}{2}\right) = \pm 0.426 P.$$

$$g) w'(0) = 0.0001 \frac{q l^2}{EI}; M(l) = 0.056 q l^2;$$

$$N(0) = -0.0729 q l; N(l) = 0.357 q l.$$

$$h) M(0) = 0.0292 q l^2; M(l) = 0.0457 q l^2;$$

$$N(0) = -0.1311 q l; N(l) = 0.329 q l.$$

$$79. w_{max} = \frac{q}{k} [1 - e^{-u} \cos u]; u = \sqrt{\frac{k}{4EI}}; u = \frac{l}{2} a.$$

$$80. I = \frac{N}{2a^3 EI} (1 + al) + \frac{M}{2a^3 EI} (1 + 2al) + \frac{q l^3}{8EI} \left(1 - \frac{8R_0}{3al}\right) - \frac{R_0}{K},$$

$$\text{where } N = q l - R_0; M = \frac{q l^2}{2} - R_0 l; a = \sqrt{\frac{k}{4EI}}.$$

$$81. w_0 = \frac{\frac{q}{k} [1 - \varphi_1(u)] + \frac{q_0 a^3 l}{24EI} \eta_1(u)}{1 + \frac{k a^3 l}{24EI} \eta_1(u)};$$

$$M_{00} = -\frac{q a^3}{3} \chi_3(u) - \frac{(q_0 l - k w_0) a}{4} \lambda_1(u).$$

where  $u = a \sqrt{\frac{k}{4EI}}$ , and  $\varphi_1(u)$ ,  $\eta_1(u)$ ,  $\chi_3(u)$  and  $\lambda_1(u)$  are I. G. Bubnov's functions for beams which lie on an elastic base.

82. The elastic line of the pontoon is symmetrical relative to the middle cross section. At the position of the coordinate axes indicated in Fig. 71, the displacement of the pontoon's

section, counting from the equilibrium position in the absence of force P, is determined by the expression

$$w(x) = \frac{Pl^3}{32\sqrt{2}EIh^3} \left[ V_2(ax) + \frac{V_1(u)V_2(u) + V_0^2(u)}{V_1(u)V_2(u) + V_1(u)V_0(u)} V_0(ax) + \right. \\ \left. + \frac{V_0(u)V_2(u) - V_1^2(u)}{V_0(u)V_2(u) + V_1(u)V_0(u)} V_2(ax) \right].$$

The bending moment in the middle cross section

$$M_{x=\frac{l}{2}} = \frac{Pl}{4} \frac{1}{\sqrt{2}u} \frac{V_0(u)V_2(u) - V_1^2(u)}{V_1(u)(V_0(u) + V_2(u)V_2(u))},$$

where  $u = \frac{l}{2}a$ ;  $a = \sqrt{\frac{By}{4EI}}$ ;  $\gamma$  is the specific gravity of water.

$$83. \quad \sigma = \sqrt{\frac{3}{1-\nu^2}} \frac{qR}{h} e^{-ax} (\cos ax - \sin ax); \quad a = \frac{5\pi}{2\sqrt{3(1-\nu^2)}} h \sqrt{\frac{R}{100h}} \approx 6.1h \sqrt{\frac{R}{100h}},$$

where

$$a = \sqrt[4]{\frac{3(1-\nu^2)}{R^3h^3}}.$$

$$84. \quad F = \frac{hR}{5\sqrt{3(1-\nu^2)}} \sqrt{\frac{100h}{R}} \approx 0.155hR \sqrt{\frac{100h}{R}}.$$

$$85. \quad x = \frac{1 + \frac{q_1}{q} \left(\frac{l_1}{l}\right)^3 \frac{l}{l_1} \frac{\psi_2(u_1)}{\psi_2(u)}}{1 + \frac{l_1}{l} \frac{l}{l_1} \frac{2\psi_0(u)}{2\psi_0(u) + \psi_1(u)}}.$$

where  $\psi_i(u)$  are the Bubnov functions for beams which lie on an elastic base;  $u = \frac{l}{2} \sqrt{\frac{k}{4EI}}$ ;  $u_1 = \frac{l_1}{2} \sqrt{\frac{k_1}{4EI_1}}$ .

$$86. \quad x = \frac{1 + \frac{q_1}{q} \left(\frac{l_1}{l}\right)^3 \frac{l}{l_1} \frac{\psi_2(u_1)}{\psi_2(u)} \left[1 - \frac{\psi_1(u_1)}{2\psi_0(u_1)}\right]}{1 + \frac{l_1}{l} \frac{l}{l_1} \frac{2\psi_0(u)}{2\psi_0(u) + \psi_1(u)} \left[1 - \frac{\psi_1^2(u_1)}{4\psi_0(u_1)}\right]}.$$

$$87. \quad R = 2 \frac{q}{a}; \quad M_{on} = \frac{q}{2a^2}, \quad \text{where } a = \sqrt[4]{\frac{k}{4EI}}.$$

88. Solution. We will write the differential equation for the bending of the beam

$$EIw^{IV}(x) + kw(x) = 0 \quad (6)$$

and the boundary conditions

$$\left. \begin{aligned} x=0: w' - w'' = 0; \\ x=\frac{l}{2}: w' = 0, w = f. \end{aligned} \right\} \quad (7)$$

where  $f$  is the sagging of the end sections of the shaft. We will introduce the function of  $w_0$  which is related to the elastic line of beam  $w$  by the dependence

$$w = w_0 + f \quad (8)$$

into the deliberation.

Substituting (8) in (6) and (7), we will have

$$\left. \begin{aligned} EI w_0'''' + k w_0 = -kf, \\ x=0: w_0 = w_0' = 0; \end{aligned} \right\} \quad (9)$$

$$\left. \begin{aligned} x=\frac{l}{2}: w_0 = w_0' = 0. \end{aligned} \right\} \quad (10)$$

The elastic line of a rigidly-fixed beam loaded by an evenly distributed load of intensity  $q = -kf$  is determined by equation (9) and boundary conditions (10). Using I. G. Bubnov's solution, we will find the shear force at  $x = \frac{l}{2}$ :  $N_{\frac{l}{2}} = -\frac{kl}{2} \mu_1(u) = -\frac{Q}{2}$ . Therefore,  $I = \frac{Q}{kl \mu_1(u)}$ .

The bending moments

$$M_{x=0} = -\frac{q l^3}{24} \chi_1(u) = -\frac{kl^3}{24} \chi_1(u) = -\frac{Ql}{24} \frac{\chi_1(u)}{\mu_1(u)};$$

$$M_{x=\frac{l}{2}} = -\frac{q l^3}{12} \chi_2(u) = -\frac{Ql}{12} \frac{\chi_2(u)}{\mu_1(u)}.$$

The intensity of the reaction of the elastic base

$$\begin{aligned} r_{x=0} = kw_{x=0} = k \left[ \frac{q}{k} [1 - \varphi_1(u)] + f \right] &= -kf [1 - \varphi_1(u)] + kf = kf \varphi_1(u) = \\ &= \frac{Q}{l} \frac{\varphi_1(u)}{\mu_1(u)}; \end{aligned}$$

$$r_{x=\frac{l}{2}} = kf = \frac{Q}{l} \frac{1}{\mu_1(u)}.$$

$$89. w'(0) = -\frac{f_2}{l} \frac{8}{3} u^4 \varphi_2(u) + \frac{f_1}{l} \left[ 1 - \frac{64}{45} u^4 \rho_0(u) \right];$$

$$w'(l) = \frac{f_1}{l} \left[ 1 + \frac{64}{45} u^4 \rho_0(u) \right] + \frac{f_2}{l} \frac{8}{3} u^4 \varphi_2(u).$$

where  $u = \frac{l}{2} \sqrt{\frac{k}{4EI}}$ , and  $\pi_0(u)$  and  $\rho_0(u)$  are I. G. Bubnov's functions for beams which lie on an elastic base.

$$90. \quad \frac{(l-a)^2}{2} \mu_0(u) + a \left( \frac{a}{2} - l \right) \rho_0(u) = (l-a)(l-a),$$

where  $u = \frac{l-a}{2} \sqrt{\frac{k}{4EI}}$ , and  $\mu_0(u)$  and  $\rho_0(u)$  are I. G. Bubnov's functions.

91. Solution. The differential equation for the bending of the beam

$$[EI(x)w''(x)]' + k(w - \bar{f}) = q(x). \quad (11)$$

Since

$$EI(x)w''(x) = M(x), \quad (12)$$

where  $M(x)$  is the given bending moment, then, substituting (12) in (11) and considering that

$$w(x) = \iint_0^x \frac{M(x)}{EI(x)} dx^2 + ax + b,$$

we will have

$$\bar{f}(x) = \frac{M''(x) - q(x)}{k} + \iint_0^x \frac{M(x)}{EI(x)} dx^2 + ax + b. \quad (13)$$

where  $a$  and  $b$  are constants determined from the condition: when  $x=0$  and  $x=l$   $\bar{f}(x) = 0$ .

$$92. \quad M_0 = \frac{6EI}{l} a_0 \frac{1}{\psi_0\left(\frac{u}{2}\right)}, \quad \text{where } u = \frac{l}{2} \sqrt{\frac{k}{4EI}}.$$

$$93. \quad \frac{Q}{2} = 32 \frac{EI}{l^3} u^4 / \mu_0(u).$$

$$94. \quad \Delta w(0) = -\frac{q}{k} [1 - \gamma_0(u)] \frac{1}{1 + \frac{48EI}{l^3 \psi_0(u)}};$$

$$\Delta w\left(\pm \frac{l}{2}\right) = \pm \frac{3q}{kl} \frac{[1 - \gamma_0(u)] \gamma_0(u)}{\psi_0(u) + \frac{48EI}{l^3}}; \quad u = \frac{l}{2} \sqrt{\frac{k}{4EI}}.$$

95. Solution. We will designate the elastic line of the beam in section  $0 \leq x \leq c$  by  $w_1$  and in section  $c \leq x \leq l$ , by  $w_2 = w_1 + w_{\text{non}}$ . Then the differential equation which determines  $w_1$  will be

$$EI w_1^{IV} = 0, \quad (14)$$

and its integral

$$w_1 = Ax^3 + Bx^2 + Cx + D. \quad (15)$$

The differential equation which determines  $w_2$  will be

$$EI w_2^{IV} + kw_2 = 0. \quad (16)$$

Subtracting equation (14) from equation (16) and considering that  $w_2 = w_1 + w_{\text{non}}$ , we will obtain the differential equation for  $w_{\text{non}}$ :

$$EI w_{\text{non}}^{IV} + kw_{\text{non}} = -kw_1. \quad (17)$$

The general integral of (17) can be written in the form

$$w_{\text{non}} = D_0 V_0 [\alpha(x-c)] + D_1 V_1 [\alpha(x-c)] + D_2 V_2 [\alpha(x-c)] + D_3 V_3 [\alpha(x-c)] - [A(x-c)^3 + B(x-c)^2 + C(x-c) + D], \quad (18)$$

where  $\alpha = \sqrt[4]{\frac{k}{4EI}}$ ;  $V_i [\alpha(x-c)]$  are N. P. Pucyrevskiy's functions.

In order to determine the integration constants, we will write the following conditions: at  $x=0$   $w_1 = w_1' = 0$ ; at  $x=c$   $w_{\text{non}} = w_{\text{non}}' = w_{\text{non}}'' = w_{\text{non}}''' = 0$  and at  $x=l$   $EI w_2' = 0$ ,  $EI w_2'' = -P$ .

From these conditions we will find:

$$\left. \begin{aligned} C = D = 0; D_0 = D_1 = 0; D_2 = \frac{B}{\alpha^3}; D_3 = \frac{3A}{\sqrt[4]{2}\alpha^3}; \\ A = -\frac{P}{6EI} \frac{V_0(\alpha c')}{V_0'(\alpha c') + \sqrt[4]{2}\alpha V_3(\alpha c') + V_1(\alpha c') V_3(\alpha c')}; \\ B = \frac{P}{2EI} \frac{c + \frac{1}{\sqrt[4]{2}\alpha} V_1(\alpha c')}{V_0'(\alpha c') + \sqrt[4]{2}\alpha V_3(\alpha c') + V_1(\alpha c') V_3(\alpha c')}. \end{aligned} \right\} \quad (19)$$

The elastic line of the beam in the final form

$$w(x) = Ax^3 + Bx^2 + \left[ \frac{B}{a^2} V_1[a(x-c)] + \frac{3A}{\sqrt{2}a^2} V_1[a(x-c)] - \right. \\ \left. - A(x-c)^3 - B(x-c)^2 \right] \quad (20)$$

where A and B are determined by formulae (19).

96. Solution. The differential equations for the bending of beams I and II, respectively, are written in the form

$$\left. \begin{aligned} EI_1 w_1^{IV} + k(w_1 - w_2) &= q_1; \\ EI_2 w_2^{IV} - k(w_1 - w_2) &= -q_2. \end{aligned} \right\} \quad (21)$$

In this case,  $w_1(x)$  and  $w_2(x)$  must satisfy the following boundary conditions: at  $x=0$

$$\left. \begin{aligned} \text{at } x=l: \quad EI_1 w_1''' &= R_1; \quad EI_2 w_2''' = -R_1; \quad EI_1 w_1'' = P_1; \quad EI_2 w_2'' = -Q_1; \\ EI_1 w_1' &= R_2; \quad EI_2 w_2' = -R_2; \quad EI_1 w_1' = -P_2; \quad EI_2 w_2' = Q_2. \end{aligned} \right\} \quad (22)$$

Adding equations (21) we will find

$$EI_1 w_1^{IV} + EI_2 w_2^{IV} = q_1 - q_2. \quad (23)$$

Dividing the first equation in system (21) by  $EI_1$  and the second, by  $EI_2$  and subtracting the second equation from the first, we will have

$$EI w^{IV} + kw = \frac{l}{l_1} q_1 + \frac{l}{l_2} q_2. \quad (24)$$

where

$$I = \frac{l_1 l_2}{l_1 + l_2}, \quad w = w_1 - w_2.$$

Based on conditions (22), the boundary conditions for function  $w(x)$  at  $x=0$  are written in the form

$$\left. \begin{aligned} EI w''' &= \frac{l}{l_1} R_1 + \frac{l}{l_2} R_1; \\ EI w'' &= \frac{l}{l_1} P_1 + \frac{l}{l_2} Q_1; \end{aligned} \right\} \quad (25)$$

at  $x = l$ :

$$\left. \begin{aligned} EI w'' &= -\frac{l}{l_1} M_1 + \frac{l}{l_2} M_2 \\ EI w''' &= -\frac{l}{l_1} P_1 - \frac{l}{l_2} Q_2 \end{aligned} \right\} \quad (26)$$

Thus, equations (23) and (24) and boundary conditions (22) and (25) make it possible to determine the elastic lines of the beams in question.

97. Solution. In the case in question (see the solution to problem 96), in order to determine the elastic lines of the beams it is possible to write the following differential equations and boundary conditions

$$\left. \begin{aligned} EI_1 w_1^{IV} + EI_2 w_2^{IV} &= q_1; \\ EI w^{IV} + kw &= q_1 \frac{l}{l_1}; \end{aligned} \right\} \quad (27)$$

at  $x = \pm \frac{l}{2}$

$$w_1 = w_2 = w = 0; \quad w_1' = w_2' = w' = 0. \quad (28)$$

where  $l = \frac{l_1 l_2}{l_1 + l_2}$ ;  $w = w_1 = w_2$ .

Using I. G. Bubnov's solution, we will write

$$w(0) = \frac{q_1}{k} \frac{l}{l_1} [1 - \varphi_0(u)]. \quad (29)$$

Integrating the first equation in system (27) four times and determining the integration constants from boundary conditions (28), we will have at  $x=0$

$$EI_1 w_1 + EI_2 w_2 = -\frac{5}{384} q_1 l^3, \quad (30)$$

hence, if we take (29) into consideration

$$w_1(0) = -\frac{5}{384} \frac{q_1 l^3}{E(l_1 + l_2)} + \frac{l}{l_1 + l_2} \frac{l_2}{l_1} \frac{q_1}{k} [1 - \varphi_0(u)]. \quad (31)$$

On the basis of I. G. Bubnov's solution

$$EI w''(0) = -\frac{q l^3}{8} \frac{1}{l} \chi_0(u)$$

From the first equation in system (27) at  $x=0$ , we will have

$$EI_1 w_1''(0) + EI_2 w_2''(0) = -\frac{q l^3}{8} \quad (32)$$

From the joint consideration of (31) and (32), we will finally find

$$M_1(0) = EI_1 w_1''(0) = -\frac{q l^3}{8} \left[ \frac{1}{l_1} + \frac{1}{l_2} \chi_0(u) \right]$$

98,

$$\begin{aligned} \chi_0 &= q l^3 \frac{1}{2 [\phi_0(u) - \phi_1(u)] \frac{q_0(u)}{\mu_0(u)} + \frac{4 l^3}{8 E I} \chi_0(u)} \\ \left( \pm \frac{1}{2} \right) &= q - \frac{2 \chi_0}{l^3} \frac{\phi_0(u) - \phi_1(u)}{\mu_0(u)} \end{aligned}$$

where  $\phi_0 = \frac{1}{2} \sqrt{\frac{k}{4 E I}} \phi_0(u)$ ,  $\phi_1(u)$ ,  $\chi_0(u)$ ,  $\mu_0(u)$ ,  $q_0(u)$  are I. G. Bubnov's functions for beams which lie on an elastic base.

$$99. \quad M(0) = \frac{\frac{q l^3}{8} \frac{1}{(2u)^2} [V_1^2(2u) - V_0(2u) V_2(2u)] + \chi_0 V_0(2u)}{V_1(2u) V_0(2u) + V_0^2(2u) + \sqrt{2} \alpha l E I [V_1(2u) V_0(2u) - V_0(2u) V_2(2u)]}$$

$$100. \quad w_1 \left( \pm \frac{l}{2} \right) = \pm \frac{h l^3}{24 E I} \psi_0(u, v); \quad w'(0) = l - \frac{5 h l^3}{384 E I} \psi_0(u, v)$$

where  $\psi_0(u, v)$ ;  $\psi_0(u, v)$  are N. V. Mattes's functions;

$$\begin{aligned} 101. \quad k &= \frac{l}{2} \sqrt{\frac{k}{4 E I}}; \quad v = 4 u^3 \sqrt{\frac{E I}{G u l^3}} \\ M_{\text{cp}} &= \frac{P l}{8} \chi_0(u, v) \frac{1}{1 + \frac{1}{3} \chi_0(u, v) v^2} \\ M_{\text{cp}} &= -\frac{P l}{8} \chi_0(u, v) \left[ 1 - \frac{0.166 \frac{\chi_0(u, v)}{\chi_0(u, v)} \chi_1(u, v) v^2}{1 + \frac{1}{3} \chi_0(u, v) v^2} \right] \\ u &= \frac{l}{2} \sqrt{\frac{k}{4 E I}}; \quad v = 4 u^3 \sqrt{\frac{E I}{G u l^3}} \end{aligned}$$

where  $\chi_1(u, v)$ ;  $\chi_0(u, v)$ ;  $\chi_0(u, v)$ ;  $\chi_0(u, v)$  are N. V. Mattes's functions.

$$\begin{aligned} 102. \quad a) \quad M(0) &= -\frac{q l^3}{8} \chi_0(u, v); \quad N \left( \pm \frac{l}{2} \right) = \pm \frac{q l}{2} \epsilon_0(u, v); \\ w_1(0) &= \frac{5 q l^4}{384 E I} \psi_0(u, v); \quad w' \left( \pm \frac{l}{2} \right) = \pm \frac{q l^3}{24 E I} \psi_0(u, v). \end{aligned}$$

where  $\varphi_0(u, v)$ ,  $\chi_0(u, v)$ ,  $\xi_0(u, v)$ ,  $\psi_0(u, v)$  are N. V. Mattes's functions;

$$u = \frac{l}{2} \sqrt{\frac{k}{4EI}}; \quad v = 4u^2 \sqrt{\frac{2(1+v)l}{\omega^2}}$$

The origin of the coordinates is located halfway down the beam.

$$\begin{aligned} b) \quad M\left(\frac{l}{2}\right) &= \frac{ql^2}{12} \frac{\chi_0(u, v)}{1 + 0,256(1+v)\chi_0(u, v)v^2 + B_1}; \\ M(0) &= -\frac{ql^2}{24} \frac{\chi_1(u, v)}{1 + 0,256(1+v)\chi_0(u, v)v^2 + B_1}; \\ N\left(\pm \frac{l}{2}\right) &= \pm \frac{ql^2}{2} \frac{\xi_1(u, v)}{1 + 0,256(1+v)\chi_0(u, v)v^2 + B_1}; \\ w(0) &= \frac{q}{k} \frac{1 - f_1(u, v) + v^2 [0,256(1+v)\chi_2(u, v)]}{1 + 0,256(1+v)\chi_0(u, v)v^2 + B_1} + \\ &+ \frac{v^2 [0,128(1+v)\chi_1(u, v) + 0,77A \frac{\omega^2}{l} \xi_1(u, v)]}{1 + 0,256(1+v)\chi_0(u, v)v^2 + B_1}, \end{aligned}$$

where  $B_1 = \frac{kAl}{2} \xi_1(u, v); \quad u = \frac{l}{2} \sqrt{\frac{k}{4EI}}; \quad v = 4u^2 \sqrt{\frac{2(1+v)l}{\omega^2}};$   
 $\chi_1(u, v) \dots \xi_1(u, v)$

are N. V. Mattes's functions.

The origin of the coordinates is located halfway down the beam.

$$103. M_0 = 1,69Pa, M_1 = -0,69Pa; w_1 = 2,26 \frac{P \cdot 10^3}{Ea}; N_0 = 1,97P.$$

$$104. M_0 = 0,99Pa, M_1 = -0,41Pa; w_1 = 2,64 \frac{P \cdot 10^3}{Ea}; N_0 = 1,4P.$$

105.

$$= \frac{1}{4\gamma} \left( \frac{\pi a}{2l} \right)^4 \frac{l}{a},$$

where  $\gamma$  is the coefficient of the effect of the concentrated force applied to the beam in the main direction at the point where it intersects the stringer on bending at this point.

106. Sagging increases  $n_1$  times:

$$\eta_1 = \frac{1 - W_0\left(\frac{\alpha}{2}\right)}{1 - W_0\left(\frac{1}{2}\right)}$$

where  $\alpha = \sqrt{\frac{k}{4EI}} = \frac{1}{l} \sqrt{\frac{l}{4\gamma} \frac{l}{a}}$ ;  $W_0(\alpha x)$  are G. V. Klishevich's functions.

$$107. \quad M_{\max} = -\frac{Pl}{16} \frac{1}{l} \left(\frac{l}{l}\right)^3 \eta_1(u); \quad u = \sqrt{\frac{3}{4} \frac{l}{l} \left(\frac{l}{l}\right)^3 \frac{l}{a}}$$

where  $\eta_1(u)$  is I. G. Bubnov's function for beams on an elastic base.

$$\begin{aligned} & \times 10^{-3} qL^3. \\ & a) \quad m(0,5l) = -2,1 \cdot 10^{-3} qL^3; \quad M(0,5l) = -15,4 \cdot 10^{-3} qL^3; \quad R = 9,5 \times \\ & b) \quad m(0) = 2,16 \cdot 10^{-3} qL^3, \quad m(0,5l) = -1,31 \cdot 10^{-3} qL^3, \quad m(l) = 3,18 \cdot 10^{-3} qL^3; \\ & \quad M(0,5l) = -4,65 \cdot 10^{-3} qL^3; \quad R = 2 \cdot 10^{-3} qL. \\ & c) \quad m(0,4l) = -14,6 \cdot 10^{-3} PL, \quad m(l) = 14,2 \cdot 10^{-3} PL; \quad M(0) = M(l) = 0,151 PL, \\ & \quad M(0,5l) = -0,0735 PL; \quad R = -0,107 P. \\ & d) \quad m(0,5l) = -0,205 PL, \quad M(0,5l) = -0,15 PL; \quad R = -0,1 P. \\ & e) \quad m(0,4l) = -27,4 \cdot 10^{-3} qL^3; \quad M(0) = M(l) = 0,00465 qL^3, \quad M(0,5l) = \\ & \quad = -2,24 \cdot 10^{-3} qL^3; \quad R = 5,2 \cdot 10^{-3} qL^3. \end{aligned}$$

Reaction  $R$  is positive in these answers if the cross connection supports the middle beam in the main direction.

109. Solution. When a beam reinforced in the main direction is present in the covering, the calculation of the cross connection is reduced to calculating a beam which lies on an elastic base and which is supported by an elastic support with rigidity

$K = \frac{(m-1)EI}{\gamma l^3}$  [see formula 3.6] at the point where it intersects the reinforced beam. For a cross connection which is rigidly fastened at the ends, the reaction of the elastic support is determined from the equation

$$\frac{\bar{q}}{k} [1 - \varphi_1(u)] - \frac{R_1 L^3}{192EI} \eta_1(u) = \frac{R_1}{K},$$

where  $\bar{q} = \frac{P}{\gamma} q$  [see formula (3.2) at  $Q=qal$ ];  $u = \frac{L}{2} \sqrt{\frac{l}{4a/l^3}}$ ;  $\varphi_1(u)$ ,  $\eta_1(u)$

are I. G. Bubnov's functions for beams on an elastic base;  $\beta$ ,  $\gamma$  are the coefficients of the effect  $(\gamma = \frac{1}{48}, \beta = \frac{5}{384})$ ;  $k = \frac{EI}{\gamma al}$  is the rigidity of the elastic base. Hence

$$R_2 = \frac{5}{2} qal \left( \frac{l}{L} \right)^3 \frac{1}{l} \frac{(m-1)(1-\eta_1(u))}{(m-1)\eta_1(u) + 4l \left( \frac{l}{L} \right)^3}.$$

Since the reaction of the middle beam in the main direction is equal to  $R_1 = q\eta_1(u)$ , at  $m=1$ , the total reaction of the middle beam in the main direction is determined by the expression

$$R = R_1 + R_2 = \frac{5}{2} qal \left[ \eta_1(u) + 4 \left( \frac{l}{L} \right)^3 \frac{1}{l} \frac{(m-1)(1-\eta_1(u))}{(m-1)\eta_1(u) + 4l \left( \frac{l}{L} \right)^3} \right].$$

$$110. \frac{l}{L} = 2.19.$$

$$111. M_{\text{max}} = \frac{Ql}{12} \frac{128 - 27a}{128 + 108a}, \text{ where } a = \left( \frac{l}{L} \right)^3 \frac{1}{l}.$$

$$112. l = \frac{64}{27} \left( \frac{L}{l} \right)^3 l.$$

113. Instructions. The presence of reinforced beams which are rigidly fixed at one end in the covering can be taken into consideration by loading the cross connection with additional forces  $P_1$  and installing additional supports with rigidity  $K$  at the point of intersection with the reinforced beams. In the case of a large number of reinforced beams, forces  $P_1$  and rigidities  $K$  are spread to a length of  $2a$ .

The moments in the cross connection  $M_x = 0.203QL$ ;  $M_{\text{max}} = -0.06QL$ .

The moments halfway down in ordinary and reinforced middle beams in the main direction  $m_x = -0.029QL$ ;  $m_{\text{max}} = -0.10QL$ , where  $Q = \frac{qal}{2}$ .

114. The moments in the cross connection:

$$M \left( x = \frac{L}{2} \right) = M(x=0) = 0.123QL; M \left( x = \frac{L}{4} \right) = -0.060QL.$$

The greatest bending moment in the middle beam in the main direction will be halfway down (on the pillar);  $m_{\text{max}} = \frac{Ql}{32}$ , i.e.  $Q = \frac{qal}{2}$ .

115. Instructions. The unknown reaction of the pillar is determined from the condition of the evenness of sagging of the cross connection due to the pillar's pressure:

$$R_1 = \frac{1.66Q}{0.563 + \frac{192Hl}{\pi l^3}}$$

116. Instructions. The calculation of the cross connection is reduced to calculating a beam on an elastic base upon which an elastic support with negative rigidity  $-\frac{EI}{\gamma l^3}$  is installed halfway down.

The moment in the span  $M(0) = -0.154QL$  and the moment in the fixing  $M(\pm \frac{L}{2}) = 0.305QL$ , where  $Q = q_0 la$ .

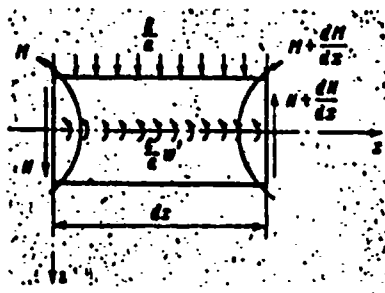


Fig. 232.

117. The moments in the cross connection of the upper covering:  $M_s = -0.033QL$ ;  $M_{sp} = 0.061QL$ . The moment half-way down the middle beam in the main direction of the upper covering  $m_{max} = 0.142QL$ . The moment in the cross connection of the lower covering:  $M_s = -0.24QL$ ;  $M_{sp} = 0.047QL$ . The moment half-way down the middle beam in the main direction of the lower covering  $m_{max} = -0.06QL$ , where  $Q = q_0 al$ .

118. Solution. Reaction intensity  $\frac{R}{a}$  and bending moment intensity  $\frac{E}{a} w''(x)$ , act on the cross connection in the case of a large number of beams in the main direction, where  $R$  is the vertical reaction at the intersection point of the cross connection with the beams in the main direction;  $w'(x)$  is the angle of rotation of the cross connection which coincides with the beam in the main direction.

We will write the equation for the equilibrium of an element in the cross connection which has length  $dx$  (Fig. 232) in order to derive the differential equation for the sagging of the cross connection. Equating the projections of all the forces on axis

Oz to zero, as well as the sum of its moments, we will have:

$$\frac{dN}{dx} = \frac{R}{a}; \quad \frac{dM}{dx} = N + \frac{c}{a} w'. \quad (33)$$

hence

$$\frac{d^2 M(x)}{dx^2} = \frac{R(x)}{a} + \frac{c}{a} w''(x). \quad (34)$$

Since  $EI w^{IV} = \frac{d^2 M}{dx^2}$ , we will obtain the differential equation for the sagging of the cross connection in the form

$$EI w^{IV}(x) - \frac{c}{a} w''(x) = \frac{R(x)}{a}. \quad (35)$$

The sagging of the beam in the main direction at the point of intersection with the cross connection

$$w = \beta \frac{Q(x) l^3}{EI} - \gamma \frac{R l^3}{EI}, \quad (36)$$

where  $Q(x)$  is the load on the beam in the main direction;  $\beta$  and  $\gamma$  are the coefficients of the effect. Eliminating reaction  $R$  from equations (35) and (36), we will have

$$EI w^{IV}(x) - \frac{c}{a} w''(x) + \frac{EI}{\gamma a l^3} w(x) = \frac{\beta}{\gamma} \frac{Q(x)}{a}. \quad (37)$$

For a freely supported cross connection, it is possible to distort the solution to equation (37) in the form

$$w(x) = \sum_{n=1}^{\infty} a_n \sin \frac{n\pi x}{L}. \quad (38)$$

Expanding load  $Q(x)$  into a series of sines, we will write

$$Q(x) = \sum_n Q_n \sin \frac{n\pi x}{L}, \quad (39)$$

where

$$Q_n = \frac{2}{L} \int_0^L Q(x) \sin \frac{n\pi x}{L} dx.$$

Substituting (38) and (39) in (37), we will have

$$a_n = \frac{\beta}{\gamma_n} \frac{Q_n}{EI \left(\frac{n\pi}{L}\right)^4 + \frac{c}{a} \left(\frac{n\pi}{L}\right)^3 + \frac{EI}{\gamma_n^3}}.$$

Consequently,

$$w(x) = \frac{\beta}{\gamma_n} \sum_{n=1}^{\infty} \frac{Q_n}{EI \left(\frac{n\pi}{L}\right)^4 + \frac{c}{a} \left(\frac{n\pi}{L}\right)^3 + \frac{EI}{\gamma_n^3}} \sin \frac{n\pi x}{L}.$$

119. Solution. Calculating the Covering by the Method of Main Bends without Consideration of Shear Deformations. Using appendix VIII in dependence on parameters  $\frac{I_n}{I_c}$  and  $\frac{c}{L}$ , let us determine the characteristic numbers  $\lambda_1$  and  $\lambda_2$ , the forms of the main bends  $v_{12}$  and  $v_{21}$  and the intensities of distributed loads  $q_1$  and  $q_2$ . In this case, when  $\frac{I_n}{I_c} = 1.25$ ,  $\frac{c}{L} = 0.25$  we will find:

$$\begin{aligned} v_{11} &= 1; v_{21} = 1.08; v_{12} = -0.7048; v_{22} = 1; \\ \lambda_1 &= \frac{x_1}{384} = \frac{17.8}{384}; \lambda_2 = \frac{x_2}{384} = \frac{0.2445}{384}; \\ q_1 &= 0.1775 \frac{Q}{a}; q_2 = 0.086 \frac{Q}{a}. \end{aligned}$$

The load on the beam in the main direction

$$Q = qaL = 0.08qL^2.$$

The rigidity factors of the elastic base:

$$k_1 = 211 \frac{EI_n}{L^3}; k_2 = 15345 \frac{EI_n}{L^3}.$$

The argument of the elastic base

$$u_1 = \sqrt[4]{\frac{k_1 L^4}{64EI_c}} = 1.423; u_2 = \sqrt[4]{\frac{k_2 L^4}{64EI_c}} = 4.16.$$

We find the values of I. G. Bubnov's function according to appendix VI:

Argument	$\varphi_1(u)$	$\chi_1(u)$	$\chi_2(u)$	$\mu_1(u)$
$\mu_1 = 1.42$ $\mu_2 = 4.16$	0.554 -0.043	-0.631 -0.0016	0.692 0.067	0.758 0.34

We will determine the sagging in the middle of the vertical stabilizer from the formula

$$w_n = \frac{I_0}{0.5J_n} (v_{11}p_1 + v_{12}p_2) = 476 \cdot 10^{-6} \frac{qL^3}{EI_n};$$

$$p_1 = \frac{q_1}{k_1} [1 - \varphi_1(\mu_1)] = 300 \cdot 10^{-6} \frac{qL^3}{EI_n};$$

$$p_2 = \frac{q_2}{k_2} [1 - \varphi_2(\mu_2)] = 4.7 \cdot 10^{-6} \frac{qL^3}{EI_n}.$$

The bending in the middle of the stringer

$$w_c = v_{21}p_1 + v_{22}p_2 = 328.7 \cdot 10^{-6} \frac{qL^3}{EI_n}.$$

The bending moments in the middle of the vertical stabilizer and stringer span:

$$M_{n, cp} = -2 \frac{QL^3}{24a} \left[ \frac{v_{11}q_1a}{Q} \chi_1(\mu_1) + \frac{v_{12}q_2a}{Q} \chi_1(\mu_2) \right] = -0.0074qL^3;$$

$$M_{c, cp} = -\frac{QL^3}{24a} \left[ \frac{v_{21}q_1a}{Q} \chi_1(\mu_1) + \frac{v_{22}q_2a}{Q} \chi_1(\mu_2) \right] = -0.00399qL^3.$$

The bending moments in the support section of the vertical stabilizer and the stringer:

$$M_{n, on} = 2 \frac{QL^3}{12a} \left[ \frac{v_{11}q_1a}{Q} \chi_2(\mu_1) + \frac{v_{12}q_2a}{Q} \chi_2(\mu_2) \right] = 0.0156qL^3;$$

$$M_{c, on} = \frac{QL^3}{12a} \left[ \frac{v_{21}q_1a}{Q} \chi_2(\mu_1) + \frac{v_{22}q_2a}{Q} \chi_2(\mu_2) \right] = 0.00332qL^3.$$

The shear forces in the support section of the vertical stabilizer and the stringer:

$$N_{n, on} = 2 \frac{QL}{2a} \left[ \frac{v_{11}q_1a}{Q} \mu_1(\mu_1) + \frac{v_{12}q_2a}{Q} \mu_1(\mu_2) \right] = 0.096qL^2;$$

$$N_{c, on} = \frac{QL}{2a} \left[ \frac{v_{21}q_1a}{Q} \mu_1(\mu_1) + \frac{v_{22}q_2a}{Q} \mu_1(\mu_2) \right] = 0.0664qL^2.$$

II. Calculating the Covering According to the Method of Main Bends with Consideration of Shear in the Walls. With consideration of shear, the coefficients of effect  $\beta_1$ ,  $\beta_2$ ,  $\gamma_{11}$ ,  $\gamma_{12}$  and  $\gamma_{22}$  for beams in the main direction have the following values:

$$\beta_1 = \frac{5}{384} \left( 1 + 25 \frac{l_0}{\omega_0^2 h} \right) = 0.0178; \beta_2 = \frac{57}{34 \cdot 255} \left( 1 + 27.3 \frac{l_0}{\omega_0^2 h} \right) = 0.013;$$

$$\gamma_{11} = \frac{1}{24} \left( 1 + 31.2 \frac{l_0}{\omega_0^2 h} \right) = 0.0608;$$

$$\gamma_{22} = \gamma_{11} = \frac{11}{384} \left( 1 + 22.7 \frac{l_0}{\omega_0^2 h} \right) = 0.0382;$$

$$\gamma_{12} = \frac{1}{48} \left( 1 + 31.2 \frac{l_0}{\omega_0^2 h} \right) = 0.0304.$$

We will take the moment of inertia  $I_0 = 0.5I_n$ ;  $I_1 = 0.5I_n$ ;  $I_2 = I_n$ .

The roots of the characteristic determinant found according to formula (3.16) are equal to:  $\lambda_1 = 0.1034$ ;  $\lambda_2 = 0.0060$ . The forms of the main bends determined according to formula (3.17) are:

$$v_{11} = 1; v_{21} = 1.116; v_{22} = 1; v_{12} = -0.697.$$

The verification of the orthogonality condition:

$$\frac{I_0}{I_1} v_{11} v_{22} + \frac{I_0}{I_2} v_{21} v_{12} = -0.697 + \frac{1.116}{1.5} = 0.$$

The intensity of the distributed load in the first and second main bends, calculated by formula (3.15):

$$q_1 = \frac{Q}{a} \frac{(v_{11} \beta_1 + v_{21} \beta_2)}{\lambda_1 \left( \frac{I_0}{I_1} v_{11}^2 + \frac{I_0}{I_2} v_{21}^2 \right)} = 0.1405 qL;$$

$$q_2 = \frac{Q}{a} \frac{(v_{12} \beta_1 + v_{22} \beta_2)}{\lambda_2 \left( \frac{I_0}{I_1} v_{12}^2 + \frac{I_0}{I_2} v_{22}^2 \right)} = 0.0717 qL.$$

The rigidity factors of the elastic base:

$$k_1 = 94.2 \frac{EI_n}{L^3}; k_2 = 1620 \frac{EI_n}{L^3}.$$

The arguments of the elastic base:

$$u_1 = \sqrt[4]{\frac{k_1 L^4}{64 EI_0}} = 1.31; u_2 = \sqrt[4]{\frac{k_2 L^4}{64 EI_0}} = 2.67.$$

The arguments which account for the effect of shear:

$$v_1 = 6.45 \cdot 10^{-3} \sqrt{\frac{L^3}{EI}} = 1.36; v_2 = 6.45 \cdot 10^{-3} \sqrt{\frac{L^3}{EI}} = 5.62.$$

We will find N. V. Mattes's function from the tables in appendix VII and the handbook (Sivertsev I. N., Davydov V. V., Mattes N. V. Students' Handbook on the Strength of Vessels with Internal Floating. M., 1950):

Argument	$f_1(u, v)$	$\chi_1(u, v)$	$\chi_2(u, v)$	$\xi_1(u, v)$
$u_1 = 1.31; v_1 = 1.36$	0.002	0.605	0.695	0.812
$u_2 = 2.67; v_2 = 5.62$	0.153	0.057	0.213	0.553

The bends in the middle of the vertical stabilizer and the stringer:

$$\begin{aligned} w_k &= v_{11} \rho_1 + v_{12} \rho_2 = 930 \cdot 10^{-6} \frac{qL^3}{EI_k}; \\ w_k &= \frac{I_0}{I_k} (v_{11} \rho_1 + v_{12} \rho_2) = 695 \cdot 10^{-6} \frac{qL^3}{EI_k}; \\ \rho_1 &= \frac{q_1}{k_1} \frac{1 - f_1(u_1, v_1) + v_1^2 \left[ \frac{1}{3} \chi_2(u_1, v_1) + \frac{1}{6} \chi_1(u_1, v_1) \right]}{1 + \frac{1}{3} \chi_2(u_1, v_1) v_1^2} = \\ &= 962 \cdot 10^{-6} \frac{qL^3}{EI_k}; \\ \rho_2 &= \frac{q_2}{k_2} \frac{1 - f_1(u_2, v_2) + v_2^2 \left[ \frac{1}{3} \chi_2(u_2, v_2) + \frac{1}{6} \chi_1(u_2, v_2) \right]}{1 + \frac{1}{3} \chi_2(u_2, v_2) v_2^2} = \\ &= 46.5 \cdot 10^{-6} \frac{qL^3}{EI_k}. \end{aligned}$$

The bending moments in the middle of the vertical stabilizer and the stringer:

$$\begin{aligned} M_{k, cp} &= 2 (v_{11} \mathcal{M}_{1, cp} + v_{12} \mathcal{M}_{2, cp}) = -0.0049 qL^2; \\ M_{c, cp} &= v_{21} \mathcal{M}_{1, cp} + v_{22} \mathcal{M}_{2, cp} = -0.0028 qL^2, \end{aligned}$$

where

$$\begin{aligned} \mathcal{M}_{1, cp} &= -\frac{q_1 L^3}{24} \frac{\chi_1(u_1, v_1)}{1 + \frac{1}{3} \chi_2(u_1, v_1) v_1^2} = -0.0298 \frac{qL^3}{12}; \\ \mathcal{M}_{2, cp} &= -\frac{q_2 L^3}{24} \frac{\chi_1(u_2, v_2)}{1 + \frac{1}{3} \chi_2(u_2, v_2) v_2^2} = -0.00063 \frac{qL^3}{12}. \end{aligned}$$

The bending moments in the supporting sections of the vertical stabilizer and the stringer:

$$M_{u, \text{en}} = 2(v_{11} M_{1, \text{en}} + v_{12} M_{2, \text{en}}) = 0.0106 q L^3$$

$$M_{c, \text{en}} = v_{21} M_{1, \text{en}} + v_{22} M_{2, \text{en}} = 0.00676 q L^3$$

where

$$M_{1, \text{en}} = \frac{q_1 L^3}{12} \frac{\chi_2(u_1, v_1)}{1 + \frac{1}{3} \chi_2(u_1, v_1) v_1^2} = 0.0685 \frac{q L^3}{12}$$

$$M_{2, \text{en}} = \frac{q_2 L^3}{12} \frac{\chi_2(u_2, v_2)}{1 + \frac{1}{3} \chi_2(u_2, v_2) v_2^2} = 0.0047 \frac{q L^3}{12}$$

The shear forces in the supporting sections of the vertical stabilizer and the stringer:

$$N_u = 2(v_{11} N_{1, \text{en}} + v_{12} N_{2, \text{en}}) = 0.0714 q L^2$$

$$N_c = (v_{21} N_{1, \text{en}} + v_{22} N_{2, \text{en}}) = 0.0507 q L^2$$

where

$$N_{1, \text{en}} = \frac{q_1 L}{2} \frac{\xi_1(u_1, v_1)}{1 + \frac{1}{3} \chi_2(u_1, v_1) v_1^2} = 0.04 q L^2$$

$$N_{2, \text{en}} = \frac{q_2 L}{2} \frac{\xi_1(u_2, v_2)}{1 + \frac{1}{3} \chi_2(u_2, v_2) v_2^2} = 0.0061 q L^2$$

A summary of the results obtained is given below:

Version	Bending moments in middle of span		Bending moments in support section	
	Vertical stabilizer	Stringer	Vertical stabilizer	Stringer
I	$-7.4 \cdot 10^{-3} q L^3$	$-4 \cdot 10^{-3} q L^3$	$15.6 \cdot 10^{-3} q L^3$	$9.3 \cdot 10^{-3} q L^3$
II	$-4.5 \cdot 10^{-3} q L^3$	$-2.8 \cdot 10^{-3} q L^3$	$11 \cdot 10^{-3} q L^3$	$6.7 \cdot 10^{-3} q L^3$
Version	Shear forces in support sections		Sagging in middle	
	Vertical stabilizer	Stringer	Vertical stabilizer	Stringer
I	$0.096 q L^2$	$0.066 q L^2$	$476 \cdot 10^{-3} \frac{q L^3}{EI_n}$	$328 \cdot 10^{-3} \frac{q L^3}{EI_n}$
II	$0.071 q L^2$	$0.051 q L^2$	$930 \cdot 10^{-3} \frac{q L^3}{EI_n}$	$695 \cdot 10^{-3} \frac{q L^3}{EI_n}$

120\*. Solution. As we know, the characteristic numbers  $\lambda_1$  and  $\lambda_2$ , the forms of the main bends  $\psi_{11}$ ,  $\psi_{12}$ ,  $\psi_{22}$  for given distributed loads  $q_1$  and  $q_2$  and also the arguments of the elastic base in each of the main bends  $u_1$  and  $u_2$  do not depend on the boundary conditions of the cross connections and can be determined from appendix VIII depending on values  $\frac{c}{L}$  and  $\frac{L}{h}$ .

In order to determine the elements of bending in each of the main bends, it is necessary to write out the boundary conditions at the ends of the corresponding beams which lie on an elastic base. In this case, in each main bend it suffices to determine only bends  $p_1(0)$  and  $p_2(0)$  and bending moments  $M_1^0$  and  $M_2^0$  in the supporting sections. In accordance with the condition  $A_1=0$ ;  $M_1=-\infty$  (the vertical stabilizer is freely supported);  $A_2=0$ ;  $M_2=0$  (the stringer is rigidly fixed). Using equations (3.19) and (3.21) and taking  $p_1(0)=p_1^0$  and  $p_2(0)=p_2^0$ , we will obtain six equations:

$$\begin{aligned} p_1^0 v_{11} + p_2^0 v_{12} &= 0; \quad p_1^0 v_{21} + p_2^0 v_{22} = 0; \quad M_1^0 v_{11} + M_2^0 v_{12} = 0; \quad p_1^0 v_{21} + p_2^0 v_{22} = 0; \\ p_1^0 &= \frac{q_1 L^3}{24EI} \psi_1(u_1) - \frac{M_1^0 L}{3EI} \left[ \psi_0(u_1) + \frac{\psi_1(u_1)}{2} \right]; \\ p_2^0 &= \frac{q_2 L^3}{24EI} \psi_2(u_2) - \frac{M_2^0 L}{3EI} \left[ \psi_0(u_2) + \frac{\psi_1(u_2)}{2} \right]. \end{aligned}$$

Solving these equations, we will have:

$$\begin{aligned} p_1^0 &= p_2^0 = 0; \\ M_1^0 &= \frac{q_1 L^3}{8} \frac{\psi_1(u_1) + \frac{q_2}{q_1} \frac{v_{21}}{v_{11}} \psi_2(u_2)}{\psi_0(u_1) + \frac{\psi_1(u_1)}{2} - \frac{v_{11} v_{22}}{v_{12} v_{21}} \left[ \psi_0(u_2) + \frac{\psi_1(u_2)}{2} \right]}; \\ M_2^0 &= -M_1^0 \frac{v_{11}}{v_{12}}; \\ M_1^0 \left( v_{21}^{(1)} - \frac{v_{11}}{v_{12}} v_{22}^{(2)} \right) + p_1^0 \left( v_{21}^{(1)} - \frac{v_{11}}{v_{12}} v_{22}^{(2)} \right) &= \\ &= -v_{21}^{(1)} q_1 - v_{22}^{(2)} q_2; \\ M_1^0 \left( v_{11}^{(1)} - \frac{v_{11}}{v_{12}} v_{12}^{(2)} \right) + p_1^0 \left( v_{11}^{(1)} - \frac{v_{11}}{v_{12}} v_{12}^{(2)} \right) &= \\ &= -v_{11}^{(1)} q_1 - v_{12}^{(2)} q_2; \\ M_1^0 &= -M_1^0 \frac{v_{11}}{v_{12}}; \quad p_1^0 = -p_1^0 \frac{v_{21}}{v_{12}}. \end{aligned}$$

121.

where

$$\begin{aligned} s_1^{(1)} &= -\frac{L}{3EI} \left[ \psi_0(u_1) + \frac{\psi_1(u_1)}{2} \right]; \\ s_1^{(2)} &= -\frac{L}{3EI} \left[ \psi_0(u_2) + \frac{\psi_1(u_2)}{2} \right]; \\ s_2^{(1)} &= -\frac{L^3}{24EI} \psi_2(u_1); \quad s_2^{(2)} = -\frac{L^3}{24EI} \psi_2(u_2); \\ s_3^{(1)} &= -\frac{8}{3L} u_1^2 \psi_2(u_1); \quad s_3^{(2)} = -\frac{8}{3L} u_2^2 \psi_2(u_2); \\ n_1^{(1)} &= -\frac{1}{L} [\psi_0(u_1) - \psi_1(u_1)]; \quad n_1^{(2)} = -\frac{1}{L} [\psi_0(u_2) - \psi_1(u_2)]; \\ n_2^{(1)} &= -\frac{L}{2} \mu_0(u_1); \quad n_2^{(2)} = -\frac{L}{2} \mu_0(u_2); \\ n_3^{(1)} &= -\frac{1}{2} \frac{L}{EI} \mu_0(u_1) - \frac{32EI u_1^4}{L^3} \mu_0(u_1); \\ n_3^{(2)} &= -\frac{1}{2} \frac{L}{EI} \mu_0(u_2) - \frac{32EI u_2^4}{L^3} \mu_0(u_2). \end{aligned}$$

122. The bending elements of the covering:

versions	$\frac{M_{x, on}}{qL}$	$\frac{M_{x, cp}}{qL}$	$\frac{M_{y, on}}{qL}$	$\frac{M_{y, cp}}{qL}$	$\frac{N_{x, on}}{qL^2/EI}$	$\frac{N_{x, cp}}{qL^2/EI}$	$\frac{N_{y, on}}{qL^2/EI}$	$\frac{N_{y, cp}}{qL^2/EI}$	$\frac{N_{z, on}}{qL^2/EI}$
a)	0,562	-0,271	0,367	-0,16	0	0,0145	0	0,0105	0
b)	0,562	-0,28	0,365	-0,175	-0,002	0,0205	0	0,0148	0,051
c)	0,562	-0,28	0,365	-0,239	0	0,0237	0	0,0161	0

123. Without consideration of shear

With consideration of shear

$$\begin{aligned} M_{x, on} &= 0,00315qL^3; \\ M_{x, cp} &= -0,000925qL^3; \\ N_{x, on} &= 0,028qL^3; \\ M_{y, on} &= 0,00233qL^3; \\ M_{y, cp} &= -0,00073qL^3; \\ N_{y, on} &= 0,019qL^3. \end{aligned}$$

$$\begin{aligned} M_{x, on} &= 0,00235qL^3; \\ M_{x, cp} &= -0,00055qL^3; \\ N_{x, on} &= 0,024qL^3; \\ M_{y, on} &= 0,00184qL^3; \\ M_{y, cp} &= -0,00045qL^3; \\ N_{y, on} &= 0,0173qL^3. \end{aligned}$$

124.  $M_{x, on} = 7,974qa^3; \quad M_{x, cp} = 3,606qa^3; \quad M_{y, on} = 3,351qa^3;$   
 $M_{y, cp} = -1,918qa^3.$

125.  $M_{x, on} = 8,46qa^3; \quad M_{x, cp} = -4,084qa^3; \quad M_{y, on} = 6,08qa^3;$   
 $M_{y, cp} = -3,01qa^3.$

126. The values of the bending moments:

Version	$M_{\text{on}}/PL$		$M_{\text{cp}}/PL$	
	Vertical stabilizer	Stringer	Vertical stabilizer	Stringer
a)	0,0807	0,0553	-0,1825	-0,0375
b)	0,0530	0,0237	-0,1044	-0,0156

127. Without consideration of shear

$$\begin{aligned} M_{\text{n. on}} &= 0,0116qL_1^2; \\ M_{\text{n. cp}} &= -0,00536qL_1^2; \\ M_{\text{c. on}} &= 0,00785qL_1^2; \\ M_{\text{c. cp}} &= -0,00323qL_1^2; \\ w_{\text{n. cp}} &= 0,246 \frac{qL_1^5}{EI_n}; \\ w_{\text{c. cp}} &= 0,331 \frac{qL_1^5}{EI_n}. \end{aligned}$$

With consideration of shear

$$\begin{aligned} M_{\text{n. on}} &= 0,0100qL_1^2; \\ M_{\text{n. cp}} &= -0,0043qL_1^2; \\ M_{\text{c. on}} &= 0,00396qL_1^2; \\ M_{\text{c. cp}} &= -0,00164qL_1^2; \\ w_{\text{n. cp}} &= 0,413 \frac{qL_1^5}{EI_n}; \\ w_{\text{c. cp}} &= 0,551 \frac{qL_1^5}{EI_n}. \end{aligned}$$

128.  $M_{\text{n. on}} = 0,0117qL_1^2;$   
 $M_{\text{c. on}} = 0,0083qL_1^2;$

$M_{\text{n. cp}} = -0,0057qL_1^2;$   
 $M_{\text{c. cp}} = -0,00314qL_1^2.$

129.  $M_{\text{n. on}} = 0,0131qL_1^2;$   
 $M_{\text{c. on}} = 0,0037qL_1^2;$

$M_{\text{n. cp}} = -0,0066qL_1^2;$   
 $M_{\text{c. cp}} = -0,00248qL_1^2.$

130.  $M_{\text{n. on}} = 0,0029PL^2;$   
 $M_{\text{c. on}} = 0,0016PL^2;$

$M_{\text{n. cp}} = -0,0005PL^2;$   
 $M_{\text{c. cp}} = -0,00048PL^2.$

131.  $R = 0,9 \cdot 10^4 q \cdot \frac{1}{1 + 0,65 \cdot 10^4 A}.$

132. a)  $w_A = \pm \frac{1}{384} \frac{PP^2}{EI};$

b)  $w_A = \left( P + \frac{17}{64} Q \right) \frac{l^3}{EI}, \text{ где } Q = \frac{ql}{2};$

$$c) \quad w_A = -\frac{7}{192} \frac{P l^3}{E I}$$

$$d) \quad w_A = -\frac{3}{64} \frac{M_0 l^2}{E I}$$

$$133. \quad l = \frac{2}{343} \frac{Q l^3}{E I}, \quad \text{where} \quad Q = \frac{q l}{2}$$

$$134. \quad a = \frac{1}{240} \frac{Q l^3}{E I}, \quad \text{where} \quad Q = \frac{q l}{4}$$

$$135. \quad M_{\text{on}} = 0$$

$$136. \quad a = \frac{P l^3}{12 I} \left( \frac{1 - \frac{2a}{l}}{\cos a} + \frac{6a}{l} \right) + \left( 1 - \frac{2a}{l} \right) \cos a + \frac{2a}{l}$$

where  $n$  is the ratio of the elongation of a broken beam to that of a straight beam.

$$137. \quad w = \frac{P l^3}{3 E I} \left[ \frac{c^3}{l^3} \left( \frac{l-c}{l} \right)^3 + \frac{E I}{G a l^3} \frac{l-c}{l} \cdot \frac{c}{l} \right], \quad \text{where} \quad G = \frac{E}{2(1+\nu)} \quad \text{is the shear modulus.}$$

$$138. \quad M_0 = \frac{q l^3}{8} \cdot \frac{1}{\left( 1 + \frac{3 E I}{G a l^3} \right)}$$

$$139. \quad w = \frac{P l^3}{3 E I} \left( 1 + \frac{3 E I}{G a l^3} \right)$$

140. Solution. On the basis of the theorem of the reciprocity of displacements, the bending in section  $x$  from concentrated force  $P=1$  which is applied at the end of the cantilever is equal to the bending in section  $x=l$  from a concentrated force equal to 1 which is applied in section  $x$ .

Therefore, the bending in section  $x=l$  from the load applied will be

$$w_{x=l} = \frac{q l^3}{3 E I} \int_0^l \frac{x^3}{l^3} \left( \frac{3}{2} - \frac{x}{2l} \right) dx = \frac{q l^3}{3 E I} \left( \frac{b^3 - a^3}{2 l^3} - \frac{b^4 - a^4}{8 l^4} \right)$$

$$141. \quad M_{\text{on}} = \frac{q l^3}{2} \left[ \frac{1}{2} \left( \frac{b^3 - a^3}{l^3} \right) - \frac{1}{4} \frac{b^4 - a^4}{l^4} \right]$$

$$142. \quad I = AP \frac{1 + \frac{1}{2} \left( \frac{c}{l} \right)^2 - \frac{3}{2} \frac{c}{l} + \frac{l-c}{l} \cdot \frac{32EI}{P}}{1 + \frac{32EI}{Pl} + \frac{32EI}{P}}$$

$$143. \quad a) \quad \frac{\partial V}{\partial R} = f; \quad b) \quad \frac{\partial V}{\partial M_{\text{св}}} = -\frac{l_1 + l_2}{l_1 l_2} f.$$

$$144. \quad R = \frac{7P}{6} \left[ \frac{3}{8} \frac{b^2 - a^2}{P} - \frac{b^2 - a^2}{10a^2} - \frac{d}{l} \left( \frac{b^2 - a^2}{2P} - \frac{b^2 - a^2}{8P} \right) \right].$$

145. The line of the effect on the bending moment at point B of the beam is shown in Fig. 233, where

$$v(x_1) = -0.8v_{\text{max}} \left[ \frac{x_1}{l_1} \left( 3\frac{x_1}{l_1} + 1 \right) + \left| \begin{array}{l} 4 \left( 1 - 2\frac{x_1}{l_1} \right) \\ x_1 > \frac{l_1}{2} \end{array} \right| \right],$$

$$v(x_2) = 0.6v_{\text{max}} \frac{x_2}{l_2} \left( 1 - \frac{x_2}{l_2} \right) \left( 2 - \frac{x_2}{l_2} \right), \quad 0 \leq x_1 \leq l_1;$$

$$v_{\text{max}} = \frac{5}{32} l_0.$$



Fig. 233.

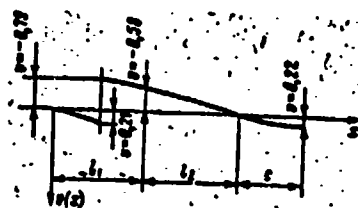


Fig. 234.

146. The line of the effect on the shear force at point B of the beam is shown in Fig. 234. The ordinates of the line of the effect in the first and second spans

$$v(x_1) = 1.047 \frac{x_1^2}{l_0^2} - 0.416 \frac{x_1^2}{l_0^2} + \left| \begin{array}{l} \left( -0.79 - 1.25 \frac{x_1^2}{l_1^2} + 0.83 \frac{x_1^2}{l_1^2} \right) \\ x_1 > \frac{l_1}{2} \end{array} \right|;$$

$$v(x_2) = -0.56 \left( 1 - \frac{x_2}{l_2} \right) + 0.134 \frac{x_2}{l_2} \left( 1 - \frac{x_2}{l_2} \right) \left( 2 - \frac{x_2}{l_2} \right).$$

In the cantilever portion of the beam  $v(x_2) = 0.22 \frac{x_2}{c}$ ;  $0 \leq x_1 \leq l_1$ ;  $l_1 \leq x_2 \leq l_1 + l_2$ ;

$$l_1 + l_2 \leq x_2 \leq l_1 + l_2 + c.$$

$$147. w(x) = \frac{Pr^3}{4EI} \cdot \frac{1}{1 + \frac{1}{4} \frac{r^3}{AEI}} \cdot \left(\frac{x}{l}\right)^3$$

$$148. w(x) = \sum_{n=2}^{\infty} a_n \varphi_n(x); \quad \varphi_n(x) = x^2 (r^{n-2} - x^{n-2});$$

$$w(x) = \frac{1}{48} \cdot \frac{Pr^3}{EI} \left(\frac{x}{l}\right)^3 \cdot \left(1 - \frac{x}{l}\right) \cdot \left(3 - 2 \frac{x}{l}\right)$$

149. The moments in fixings  $M_0 = M_1 = \frac{1}{1078} Pr^3$ . The bending in the middle of the span  $= -\frac{0.312}{384} \frac{Pr^3}{EI}$ , where  $l = \frac{1}{3} R^3$ .

150.  $M_A = 0.85Pr$ . The moment is directed along the hour hand.

151. For the assemblies shown in Fig. 117, the horizontal reaction of the support

$$H = \frac{3}{8} q^2 \frac{1 + 2\pi \frac{r}{h} + 12 \frac{r^2}{h^2} + 2\pi \frac{r^3}{h^3}}{1 + \frac{3}{2} \pi \frac{r}{h} + 6 \frac{r^2}{h^2} + \frac{3}{4} \pi \frac{r^3}{h^3}}$$

The vertical reaction of the support is  $R = qr$ ; at  $h=r$   $H=0.68$   $qr$ .

Figure 235 shows the bending moment and shear diagrams.

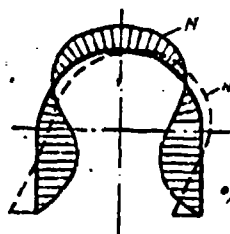


Fig. 235.

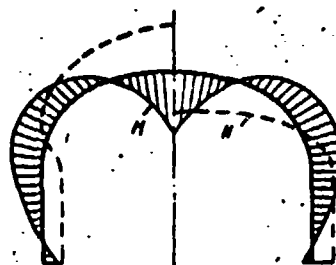


Fig. 236.

For the assemblies shown in Fig. 118, the bending moment at the point of application of pressure  $P$  is equal to  $M = -0.37Pr$ . The bending moment in the rigid fixing  $M_0 = -0.047Pr$ .

The horizontal reaction in the fixing  $H=0.2P$ . The vertical reaction in the fixing  $R=\frac{P}{2}$ . The bending moment and shear diagrams are shown in Fig. 236.

152.

$$R_1 = R_2 = \frac{P}{2}; H_1 = -H_2 = \frac{P}{2}$$

153.

$$I = \frac{2Pq^2}{EI} \cos \alpha$$

154. Solution. In order to determine the unknown angle, it is necessary to compose an expression for displacement in the direction perpendicular to that of the effect of force  $P$  and to equate this displacement to zero.

We will apply concentrated force  $P_0$  which is directed perpendicular to force  $P$  to cross section A. Then the equation of the bending moment for the beam is noted in the form:

$$M(\varphi) = (P \sin \alpha + P_0 \cos \alpha) r \sin \varphi + 2(P \cos \alpha - P_0 \sin \alpha) r \sin^2 \frac{\varphi}{2},$$

where  $r$  is the length of the arc counted off from section A.

On the basis of the Castigliano theorem, the displacement in the direction of force  $P_0$  will be

$$I = \frac{P_0}{EI} \left[ P \sin \alpha \cos \alpha \int_0^{\frac{\pi}{2}} \sin^2 \varphi d\varphi + 2P (\cos^2 \alpha - \sin^2 \alpha) \int_0^{\frac{\pi}{2}} \sin \varphi \times \sin^2 \frac{\varphi}{2} d\varphi - 2P \sin 2\alpha \int_0^{\frac{\pi}{2}} \sin^2 \frac{\varphi}{2} d\varphi \right].$$

Equating this displacement to zero, we will find  $\operatorname{tg} 2\alpha = -\frac{2}{4-\pi}$ .

$$155. \quad a) M = -\frac{Pr}{2} \left( \frac{2}{\pi} - \sin \theta \right); \quad b) M = -\frac{Pr}{2\pi} \left[ 1 + \frac{1}{2} \cos \theta - (\pi - \theta) \sin \theta \right].$$

156. The bending moments:  $M_1 = 0.0814 qr^2$ ;  $M_2 = -0.0306 qr^2$ ;  $M_3 = 0.0414 qr^2$ . The force in the spacer  $T_p = 0.712qr$ . The shear forces:  $N_1 = 0.356qr$ ;  $N_2 = 0.128qr$ ;  $N_3 = 0$ . The axial forces:  $T_1 = -1.135qr$ ;  $T_2 = -1.072qr$ ;  $T_3 = -1.144qr$  (Fig. 237).

157. The bending moments:  $M_1 = -0.223qr$ ;  $M_2 = 0.192qr$ ;  $M_3 = 0.309qr$ . The shear forces:  $N_2 = 0.59qr$ ;  $N_1 = N_3 = 0$ . The axial forces  $T_1 = -1.293qr$ ;  $T_2 = -1.50qr$ ;  $T_3 = -1.707qr$  (Fig. 238).

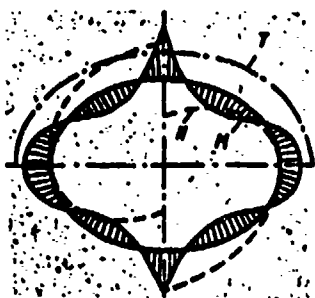


Fig. 237.

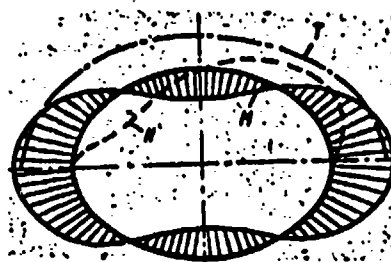


Fig. 238.

158. Solution. We will designate the angle of rotation at the origin of the coordinates of an infinite beam which lies on an elastic base and is under the effect of arbitrary load  $q(x) = q_0 \varphi(x)$  at  $q_0 = 1$  by  $\delta$ . Then it is possible to write  $\int_{-\infty}^{\infty} q(x) dx = M_0 q_0 \delta$  on the basis of the theorem of displacement reciprocity. Hence, substituting expression  $w_m$  in  $q(x)$ , we will have

$$\delta = \frac{1}{2\alpha^3 EI} \int_{-\infty}^{\infty} \varphi(x) (\cos \alpha x - \sin \alpha x) e^{-\alpha x} dx.$$

The unknown moment in the fixing is determined from equation  $w_m(0) + q_0 \delta = 0$ ; since  $w_m(0) = -\frac{M_0}{2\alpha EI}$ , then  $M_0 = 2\alpha EI q_0 \delta$ . At  $q(x) = q_0 [\varphi(x) = 1]$  (Fig. 126)

$$\delta = \frac{1}{2\alpha^3 EI} \int_{-\infty}^{\infty} e^{-\alpha x} \sin \alpha x dx$$

and, consequently,

$$M_0 = \frac{q_0}{\alpha^2} \int_{-\infty}^{\infty} e^{-\alpha x} \sin \alpha x dx$$

at  $q(x) = q_0 \frac{x}{l} [\varphi(x) = \frac{x}{l}]$  (Fig. 127)

$$\delta = \frac{1}{4\alpha^3 EI} \{ e^{-\alpha l} [(1 + 2\alpha l) \sin \alpha l + \cos \alpha l] - 1 \}$$

and, consequently,

$$M_0 = \frac{q_0}{2\alpha^2 l} \{ e^{-\alpha l} [(1 + 2\alpha l) \sin \alpha l + \cos \alpha l] - 1 \}.$$

159.

$$\delta = -\frac{QR_0 e^{-\alpha l}}{2EI} \cos \alpha l.$$

160.

$$w(0) = -\frac{Q}{k} \left( \sin \alpha l - \cos \alpha l + \frac{1}{\alpha l} \sin \alpha l \right) e^{-\alpha l}.$$

161. Instructions. Determine the bending moment in the beam by integrating the differential equation  $(EIw'')' = q - r$  and using the boundary conditions:  $x=0, EIw'' = M_0$ ;  $x=l, EIw'' = M_1$ .

The unknown moments  $M_0$  and  $M_1$  and coefficients  $r_n$  are determined from the system of equations:

$$\frac{\partial V}{\partial M_0} = 0; \quad \frac{\partial V}{\partial M_1} = 0; \quad \frac{\partial V}{\partial r_n} = 0.$$

where

$$V = -\frac{1}{2} \int_0^l \frac{M^2}{EI} dx + \frac{1}{2} \int_0^l \frac{r^2}{k} dx;$$

$$M = -\frac{Qx}{2}(l-x) + M_0 \left(1 - \frac{x}{l}\right) + M_1 \frac{x}{l} + \left(\frac{l}{\pi}\right)^2 \sum_n \frac{r_n}{n^2} \sin \frac{n\pi x}{l}$$

162. The expression for potential energy must be supplemented with the potential shear energy  $V_{cs} = \frac{1}{2} \int_0^l \frac{N^2}{G\omega} dx$ , where

$$N = -\frac{Q}{2}(l-2x) + \frac{M_1 - M_0}{l} + \frac{l}{\pi} \sum_n \frac{r_n}{n} \cos \frac{n\pi x}{l}.$$

163. Instructions. When calculating the generalized displacements of the beams in the main direction which correspond to generalized force  $r_n$ , consider the fact that bending of the beam in the main direction at the point of intersection with the cross connection can be determined from formula  $w = \beta \frac{QR}{EI} - \gamma \frac{Rl}{EI}$ , where  $R$  is the interaction reaction of the beams in both directions;  $Q$  is the load on the beam in the main direction;  $l$  is the length of the beam in the main direction;  $I$  is the moment of inertia of the beam in the main direction;  $\beta$  and  $\gamma$  are the coefficients of the effect.

When there are many beams, the potential energy from sagging of all the beams in the main direction can be computed from the formula  $V_z = \frac{1}{2} \int_0^l V_1 dx$ , where  $a$  is the distance between the beams in the main direction;  $V_1$  is the potential energy from sagging of one beam in the main direction.

The unknown moments  $M_0$  and  $M_1$  and coefficients  $r_n$  are determined from the system of equations:

$$\frac{\partial V}{\partial M_0} = 0; \quad \frac{\partial V}{\partial M_1} = 0;$$

$$\frac{\partial V}{\partial r_n} = \int_0^L \left( \beta \frac{Q^n}{EI} - \gamma \frac{a^n}{EI} \sum_j r_j \sin \frac{j\pi x}{L} \right) \sin \frac{n\pi x}{L} dx,$$

where  $V$  is the potential energy from bending of the cross connection

$$V = \frac{1}{2} \int_0^L \frac{M^2 dx}{EI} + \frac{1}{2} \int_0^L \frac{N^2 dx}{GA};$$

$$M = M_0 \left( 1 - \frac{x}{L} \right) + M_1 \frac{x}{L} - \frac{r_1^2}{2} (L-x) + \left( \frac{L}{\pi} \right)^2 \sum_j \frac{r_j}{j} \sin \frac{j\pi x}{L};$$

$$N = \frac{M_1 - M_0}{L} - \frac{r_1}{2} \left( 1 - \frac{2x}{L} \right) + \frac{L}{\pi} \sum_j \frac{r_j}{j} \cos \frac{j\pi x}{L}.$$

164.  $w(x) = \frac{P\beta^2}{6EI} \cdot \frac{x}{L} (0.068 + 0.377 \frac{x}{L}).$

165.  $w(x) = \frac{q\beta^2}{3} \frac{x^3}{4EI + \frac{4}{3}T\beta + \frac{M^2}{160} + \frac{\beta^2}{\lambda}}$

166. a)  $w(x) = \frac{2P\beta^2}{\pi^2 EI} \sum_{n=1}^{\infty} \frac{\sin \frac{n\pi c}{L}}{n^2 (1 + \frac{T}{n^2 \beta})} \sin \frac{n\pi x}{L}$ , b)  $w(x) = \frac{4q\beta^2}{\pi^2 EI} \sum_{n=1, 3, 5}^{\infty} \frac{\sin \frac{n\pi x}{L}}{n^3 (1 + \frac{T}{n^2 \beta})}$

where  $r_n = \frac{n^2 EI}{\beta}$ .

167.  $w(x) = \frac{q\beta^2}{12EI} \frac{1 - \cos \frac{2\pi x}{L}}{1 - \frac{T}{\beta}}$ , where  $T = \frac{4\pi^2 EI}{\beta}$ .

When  $T = 0.8T_0 = \left( \frac{1}{2} \right) = 0.00636 \frac{q\beta^2}{EI}$ . The precise value of sagging

$$w_1 \left( \frac{1}{2} \right) = 0.00660 \frac{q\beta^2}{EI}.$$

168.  $T_0 = \left[ EI \left( \frac{n\pi}{L} \right)^2 + k \left( \frac{1}{n\pi} \right)^2 + \frac{2\delta k}{L} \left( \frac{1}{n\pi} \right)^3 \times \left( e - \frac{1}{2n\pi} \sin \frac{2n\pi a}{L} \right) \right]_{\min}.$

169. a)  $T_0 = \frac{8}{3\pi} \frac{\pi^2 EI_0}{\beta} \approx 0.850 \frac{\pi^2 EI_0}{\beta}$ ; b)  $T_0 \approx 0.797 \frac{\pi^2 EI_0}{\beta}.$

170. Instructions. The form of the loss of stability of the rod at an elastic support rigidity which is less than the critical rigidity is a straight line which is antisymmetrical to the middle of the span

$$K_{sp} = \frac{2\pi^2 EI}{\beta}.$$

$$171. \tau_0 = \frac{\pi^2 EI}{L^2} \left[ n^2 + \frac{\gamma}{n^2} \left( \frac{l}{L} - \frac{\cos n\pi}{n\pi} \sin \frac{n\pi l}{L} \right) \right] \text{ where } \gamma = \frac{h^2}{\pi^2 EI}.$$

The number of half-waves of the form of stability loss  $n$  must be selected from the condition of obtaining the smallest value of  $T_0$ .

$$172. \tau_0 = \frac{\pi^2 EI}{L^2} \frac{l + \frac{1}{2} \frac{l_0}{L} l_0 + \frac{L}{2\pi} \left( \frac{l_0}{L} - 1 \right) \sin \frac{2\pi l}{L}}{l + \frac{1+m}{\pi} l_0 - m \frac{L}{2\pi} \sin \frac{2\pi l}{L}}.$$

If we consider the effect of deviation from Hooke's law,  $E=E'$ , where  $E'$  is the given modulus of elasticity for the portion of the beam with inertial moment of the cross section  $I$  and ratio  $\frac{l_0}{L}$  is introduced instead of  $\frac{E_0 l_0}{E L}$ , where  $E_0$  is the given modulus for the middle section of the beam.

173. Solution. Let the lower rod bend according to the law  $w_0 = A_0 \sin \frac{\pi x}{a}$ , during the loss of stability, the upper - according to the law  $w_1 = A_1 \sin \frac{\pi x}{a}$ , and the vertical rods bend according to the law  $w_2 = A_2 \sin \frac{\pi x}{b}$ . Then, from the condition of the equality of the angles of rotation at the junctions it is easy to obtain the following relationships between the coefficients  $A_0$ ,  $A_1$  and  $A_2$ :

$$A_0 \frac{\pi}{a} = A_1 \frac{\pi}{b}; \quad A_1 \frac{\pi}{b} = A_2 \frac{\pi}{a}.$$

or

$$A_1 = \frac{b}{a} A_0; \quad A_2 = \frac{a}{b} A_1 = A_0.$$

With consideration of the effect of the deviation from Hooke's law on stability, the potential energy of the deformation of the rods in the assembly are written in the form

$$V = \frac{1}{2} E_0 J_0 \int_0^a w_0'^2 dx + \frac{1}{2} E_2 J_2 \int_0^b w_2'^2 dx + E I_1 \int_0^a w_1'^2 dx.$$

The work of the compressive forces will be

$$U = \frac{1}{2} T \int_0^a w_0'^2 dx + \frac{1}{2} \lambda T \int_0^b w_2'^2 dx,$$

where  $E_0$  is the given modulus for the lower rod;  $E_2$  is the given modulus for the upper rod.

Substituting the values of bends  $w_0$ ,  $w_1$  and  $w_2$  in these expressions and integrating, we will have:

$$V = \frac{1}{4} \left[ a \left( \frac{\pi}{a} \right)^4 (E_0 J_0 + E_2 J_2) A_0^2 + \frac{1}{2} b \left( \frac{b}{a} \right)^4 E_1 \left( \frac{\pi}{b} \right)^4 A_0^2 \right]$$

$$U = \frac{T}{4} \left[ \left( \frac{\pi}{a} \right)^4 a + \lambda \left( \frac{\pi}{a} \right)^4 a \right] A_0^2$$

Since  $\frac{\partial(V-U)}{\partial A_0} = 0$ , then  $T_0 = \frac{\pi^4}{a^4} \frac{E_0 J_0 + E_2 J_2 + 2E_1 \frac{a}{b}}{1+\lambda}$

$$174. T_0 = \frac{\pi^4 EI}{b^3 \left( 1 + \frac{\pi^4 EI}{G a^4} \right)}$$

$$175. T_0 = \frac{16.54 \pi^4 EI}{b^3}$$

$$176. q_0 = 7.89 \frac{EI}{b^3}$$

$$177. T_0 = \frac{3}{2} \frac{EI_0}{b^3}$$

$$178. T_0 = 38.6 \frac{EI}{b^3}$$

$$179. T_0 = 5.2 \frac{EI}{b^3}$$

$$180. w(x) = \frac{P P_0}{(2u)^3 EI} [(1 - \cos k^2 x) (\lg 2u - k^2 x + \sin k^2 x)], \text{ where } 2u = k^2 l; M = 1.56 P l.$$

$$181. w(x) = \frac{q^N}{(2u)^3 EI} \left\{ \frac{u^3}{2} \frac{x}{l} + (1 - \operatorname{ch} u) \frac{\operatorname{sh} k^2 x}{\operatorname{sh} 2u} + \right. \\ \left. + \int_0^x \operatorname{ch} [k^2 (x - 0.5l)] - 1 - 2u^2 \left( \frac{x}{l} - 0.5 \right)^2 \right\}; w\left(\frac{l}{2}\right) = \frac{q^N}{2EI(2u)^3} I_0(u),$$

$$\text{where } I_0(u) = \frac{u^3}{2} + \frac{1}{\operatorname{ch} u} - 1; u = k^2 l/2.$$

$$182. M(0) = -\frac{EI}{l} u \frac{\sin 2u - 2u \cos 2u}{1 - \cos 2u - u \sin 2u};$$

$$M(l) = \frac{EI}{l} u \frac{2u - \sin 2u}{1 - \cos 2u - u \sin 2u}; R(0) = -R(l) =$$

$$= -\frac{2EI}{l^3} u^3 \frac{1 - \cos 2u}{1 - \cos 2u - u \sin 2u}, \text{ where } u = \frac{l}{2} \sqrt{\frac{T}{EI}}.$$

$$183. M(0) = -M(l) = \frac{2EI}{l^3} u^3 \frac{\lg u}{\lg u - u}; R(0) = -R(l) =$$

$$= \frac{4EI}{l^3} u^3 \frac{\lg u}{\lg u - u}, \text{ where } u = \frac{l}{2} \sqrt{\frac{T}{EI}}.$$

184.  $M_1 = 0.323 q l^2; M_2 = -0.338 q l^2; I = 0.138 \frac{q l^4}{EI}$

185.  $T = 11.7 \frac{EI}{l^3}$

186.  $w(x) = \left( \frac{4q l^3}{\pi^3 D} \cdot \frac{1}{1-a} + \frac{8}{3\pi} \cdot \frac{a}{1-a} \right) \sin \frac{\pi x}{l}$ , where  $a = \frac{T l^3}{\pi^3 D}$

187. Instructions. Use graphs [3, T II] to determine the stress. In the cross section  $\sigma_{1\max} = -5660 \text{ kg/cm}^2$  (in the bulkhead frame); in the meridional section -  $\sigma_{2\max} = -5420 \text{ kg/cm}^2$  (in the middle of the spacing).

188.  $2a \left( 1 - \frac{AT}{l} \right) \cos 2a - \sin 2a = 0; T_0 = 11.2 \frac{EI}{l^3}$ , where  $2a = K'l$

189.  $T_0 = 16.5 \frac{EI}{l^3}$

190. Instructions. Write the expression for the bending of the beam with consideration of a jump in the shear force at  $x=a$ . The stability equation

$$\sin K^2 a \sin K^2 (l-a) - \left[ K^2 a - \frac{AEI}{l^3} (K^2 l)^3 \right] \sin K^2 l;$$

$$T_0 = 4.54 \frac{EI}{l^3}$$

191. Instructions. The form of the stability loss will be symmetrical or antisymmetrical to the middle of the length of the beam.

$$T_0 = \frac{\pi^2 EI}{4l^3} \quad \text{at } K = \frac{1}{l} > \frac{\pi^2 EI}{4l^3};$$

$$T_0 = \frac{l}{\lambda} \quad \text{at } K = \frac{1}{\lambda} < \frac{\pi^2 EI}{4l^3};$$

$$K_{\text{cr}} = \frac{1}{\lambda_{\text{cr}}} = \frac{\pi^2 EI}{4l^3}$$

192. Instructions. The shape of the stability loss will be antisymmetric to the middle of the span  $T_0 = \frac{\pi^2 EI}{l^3}$ .

193. We will use the instructions to problem 190.  $T_0 = \frac{\pi^2 EI}{4L^2}$ .

194. The shape of the beam's stability losses will be symmetrical to its middle  $T_0 = \frac{\pi^2 EI}{L^2}$ .

$$195. \quad \frac{(Kc)^2}{3} \left(1 - \frac{c}{l}\right)^2 \left[ \frac{1}{\pi} \left(\frac{l}{c} - 1\right) + \frac{3}{Kc} \left(\frac{1}{Kc} - \frac{1}{16Kc}\right) \right] - 1 = 0.$$

At  $\kappa = \frac{1}{2}$  and  $c = \frac{l}{2}$ , the stability equation will have the form  $\left(\frac{2\pi^2}{3} - 1\right) \frac{EI}{L^2} = 0$ , where  $\kappa = \frac{\pi l}{2}$ , hence  $T_0 = \frac{14.3EI}{L^2}$ .

196. The system of differential equations which describe the neutral equilibrium of the upper and lower beams is written in the form

$$\left. \begin{aligned} EI_1 w_1'' + T w_1' + \frac{K}{\pi} (w_1 - w_2) &= 0, \\ EI_2 w_2'' + \frac{K}{\pi} (w_2 - w_1) &= 0, \end{aligned} \right\} \quad (40)$$

where  $w_1$  and  $w_2$  are the bends of these beams.

The solution to system (40) which satisfies the boundary conditions is determined in the form:

$$\left. \begin{aligned} w_1 &= a_1 \sin \frac{\pi x}{l}, \\ w_2 &= a_2 \sin \frac{\pi x}{l}. \end{aligned} \right\} \quad (41)$$

Substituting (41) in (40), we will obtain a system of homogeneous algebraic equations relative to the unknown  $a_1$  and  $a_2$ . Equating the determinant of this system to zero, we will obtain the following expression for the Euler force:

$$T_0 = \frac{\pi^2 E}{L^2} \left[ \pi^2 I_1 + I_2 \frac{1}{\pi^2 EI_1 \left(\frac{\pi}{L}\right)^2 \frac{a}{K} + \frac{1}{\pi^2}} \right],$$

where the number  $n$  must be selected from the condition of obtaining the smallest value of  $T_0$ .

197. Instructions. In order to find the Euler load, one should write the structure's equilibrium equation for the deviating position, considering that the angle between the rods remain right angles in the deviating position too. Then for the structure shown in Fig. 151,  $P_0 = \frac{EI}{2A}$ , and for the structure shown in Fig. 152,  $P_0 = \frac{6EI}{18}$ .

198.  $T_0 = 13,2 \frac{EI}{l^3}$ .

199.  $T_0 = 43,4 \frac{EI}{l^3}$ .

200.  $T_0 = \frac{\pi^2 EI}{l^3} \left[ \frac{n^2}{1 + \frac{(n\pi)^2 EI}{G A l^2}} + \frac{1}{n^2} \frac{A^2}{\pi^2 EI} \right]_{\min}$ ; with consideration of shear  
 $T_0 = 3,85 \frac{\pi^2 EI}{l^3}$ ;  $n = 4$ ; without consideration of shear,  $T_0 = 6,5 \frac{\pi^2 EI}{l^3}$ ;  $n = 2$ .

201.  $l = 4,86 \frac{EI}{T_0} \left( \frac{l}{L} \right)^2$ .

202.  $l = \frac{2}{3} \pi^2 \frac{EI}{T_0} \left( \frac{l}{L} \right)^2$ .

203.  $l \approx 0,4l$ .

204.  $i_1 = 0,545l$ ;  $T_0 = \frac{4\pi^2 EI}{L^3}$  at  $i_1 \geq 0,545l$ . At  $i_1 < 0,545l$ ,  $T_0$  is determined from the equation

$$\frac{\pi^2 L}{2} - \frac{245l}{\left(\frac{L}{2}\right)^3} \cdot \left(\frac{\pi^2 L}{2}\right)^3 = 18 \frac{\pi^2 L}{2}, \quad \text{where } A = \frac{0,166L^3}{48EI_1}.$$

205.  $T_0 = K \frac{l_1 l_2}{l_1 + l_2}$ .

206.  $T_0 = 6,5 \frac{EI}{l_1^3}$ .

207.  $T_0 = 10,05 \frac{EI}{l_1^3}$ .

208.  $T_0 = 1,74 \frac{EI}{l_1^3}$ .

209. Three supports.

210.  $\sigma_{sp} = 2700 \text{ kg/cm}^2$ .

211.  $\tau_s = 14,4 \frac{\pi^2 EI}{l^3}$ . During "spreading"  $\tau_s = 14,46 \frac{\pi^2 EI}{l^3}$ .

212.  $J_s = 36 \frac{\pi^2 EI}{l^3}$ . During "spreading"  $\tau_s = 44,6 \frac{\pi^2 EI}{l^3}$ .

213.  $\sigma_{sp} = 2400 \text{ kg/cm}^2$ .

214.  $\kappa_1 = \kappa_2 = 0.72$ .

215.  $I = 1,015 \cdot 10^{-4} \text{ m}^4$ ;  $I = 12,6 \cdot 10^{-4} \text{ m}^4$ .

216.  $I = 3,1 \cdot 10^{-4} \text{ m}^4$ ;  $I_m = 7,3 \cdot 10^{-4} \text{ m}^4$ .

217.  $I = 4,13 \cdot 10^{-4} \text{ m}^4$ ; ; for a covering without carlings  
 $I = 22,7 \cdot 10^{-4} \text{ m}^4$ .

218.  $\sigma_{sp} = 3600 \text{ kg/cm}^2$ .

**Table.**

[illegible]

Table (cont'd).

9	948	$\pm 0.466$	$\mp 0.770$	$\pm 0.441$	431	$\mp 0.884$	$\mp 0.847$	$\pm 0.326$	-0.00	$\mp 0.017$	$\pm 0.814$	$\pm 0.308$	$\frac{388}{175}$	59.4
10	859	$\pm 0.332$	$\pm 0.233$	$\mp 0.905$	590	$\mp 0.745$	$\pm 0.372$	$\pm 0.555$	-1.69	$\pm 0.037$	$\pm 0.545$	$\pm 0.340$	$\frac{435.1}{381.5}$	399
11	2347	$\pm 0.946$	$\pm 0.338$	$\mp 0.413$	191	$\pm 0.740$	$\pm 0.873$	$\pm 0.577$	-0.43	$\pm 0.388$	$\mp 0.640$	$\pm 0.380$	$\frac{1281}{1408}$	3499
12	834.7	$\pm 0.770$	$\pm 0.550$	$\pm 0.310$	140.8	$\mp 0.920$	$\pm 0.330$	$\mp 0.320$	-0.94.9	$\pm 0.188$	$\pm 0.889$	$\mp 0.789$	$\frac{551.3}{654.5}$	473.3
13	472	$\pm 0.255$	$\pm 0.796$	$\pm 0.548$	624	$\pm 0.950$	$\pm 0.479$	$\mp 0.829$	-1.49	$\mp 0.878$	$\pm 0.351$	$\mp 0.188$	$\frac{673}{916}$	683.3
14	420	$\pm 0.778$	$\pm 0.488$	$\pm 0.399$	183	$\mp 0.889$	$\pm 0.111$	$\pm 0.856$	-3.3	$\mp 0.373$	$\pm 0.887$	$\mp 0.353$	$\frac{173}{113}$	46
15	601.6	$\pm 0.652$	$\pm 0.555$	$\pm 0.502$	254.3	$\mp 0.774$	$\pm 0.308$	$\pm 0.503$	-53.3	$\mp 0.888$	$\pm 0.726$	$\mp 0.888$	$\frac{288.5}{327.5}$	383.3
16	661.0	$\pm 0.811$	$\pm 0.491$	$\pm 0.350$	300	$\mp 0.436$	$\pm 0.638$	$\pm 0.639$	-81.5	$\mp 0.004$	$\pm 0.886$	$\mp 0.781$	$\frac{388}{381}$	134.1
17	584	$\pm 0.740$	$\pm 0.490$	$\pm 0.470$	100	$\mp 0.430$	$\pm 0.880$	$\mp 0.328$	-04	$\pm 0.589$	$\pm 0.888$	$\mp 0.888$	$\frac{381}{381}$	88
18	586.9	$\pm 0.057$	$\pm 0.886$	$\pm 0.440$	132.3	$\pm 0.448$	$\mp 0.787$	$\pm 0.433$	-819.1	$\mp 0.888$	$\pm 0.881$	$\mp 0.788$	$\frac{388}{388}$	783.3